Unit 7

THE PYRAMIDS AT GIZA IN EGYPT ARE AMONG THE BEST KNOWN PIECES OF ARCHITECTURE IN THE WORLD. THE PYRAMID OF KHAFRE WAS BUILT AS THE FINAL RESTING PLACE OF THE PHARAOH KHAFRE AND IS ABOUT 136 M HIGH.

MEASUREMENT

Unit Outcomes:

After completing this unit, you should be able to:

- *solve problems involving surface area and volume of solid figures.*
- *know basic facts about frustums of cones and pyramids.*

Main Contents

INTRODUCTION

RECALL THAT GEOMETRICAL FIGURES THAT HAVE THREE DIMENSIONS (LENGTH, WIDTH CALLED solid figures. FOR EXAMPLE, CUBES, PRISMS, CYLINDERS, CONES AND PYRAMIDS THREE DIMENSIONAL SOLID FIGURES. IN YOUR LOWER GRADES, YOU HAVE LEARNT HOW SURFACE AREAS AND VOLUMES OF SOLID FIGURES LIKE CYLINDERS AND PRISMS. IN T WILL LEARN MORE ABOUT SURFACE AREAS AND VOLUMES OF OTHER SOLID FIGURES. STUDY ABOUT SURFACE AREAS AND VOLUMES OF COMPOSED SOLIDS AND FRUSTUMS AND CONES.

OPENING PROBLEM

ATO NIGATU DECIDED TO BUILD A GARAGE AND BEGAN BY CALCULATING THE NUM REQUIRED. THE FLOOR OF THE GARAGE IS RECTANGULAR WITH LENGTHS 6 M AND 4 M. T THE BUILDING IS 4 M. EACH BRICK USED TO CONSTRUCT THE BUILDING MEASURES 22 CM BY 7 CM.

A HOW MANY BRICKS MIGHT BE NEEDED TO CONSTRUCT THE GARAGE?

B FIND THE AREA OF EACH SIDE OF THE BUILDING.

C WHAT MORE INFORMATION DO YOU NEED TO FIND THE EXACT NUMBER OF BRI REQUIRED?

7.1 REVISION ON SURFACE AREAS AND VOLUMES OF PRISMS AND CYLINDERS

THERE ARE MANY THINGS AROUND US WHICH ARE EITHER prismatic OR cylindrical IN SHAPE. IN THIS SUB-UNIT, YOU WILL CLOSELY LOOK AT THE GEOMETRIC SOLIDS CALLED prisms AND cylinders AND THEIR SURFACE AREAS AND VOLUMES.

LET E_1 AND E_2 BE TWO PARALLEL PLANES, ℓ A LINE INTERSECTING BOTH PLANES, AND R A REGION IN E_1. FOR EACH POINT P OF R, LET P' BE THE POINT IN E_2 SUCH THAT $\overline{PP'}$ IS PARALLEL TO ℓ.

THE UNION OF ALL POINTS P' IS A REGION R' IN E_2 CORRESPONDING TO THE REGION R IN E_1. THE UNION OF ALL THE SEGMENTS $\overline{PP'}$ IS CALLED A solid region *D*. THIS SOLID REGION IS KNOWN AS A cylinder. SEE FIGURE 7.2

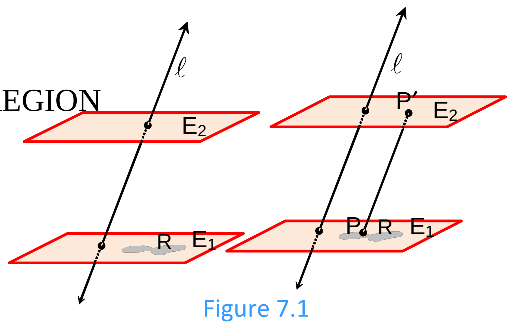


Figure 7.1

Some important terms

For the cylinder D, the region R is called its lower base or simply **base** and R' is its **upper base**.

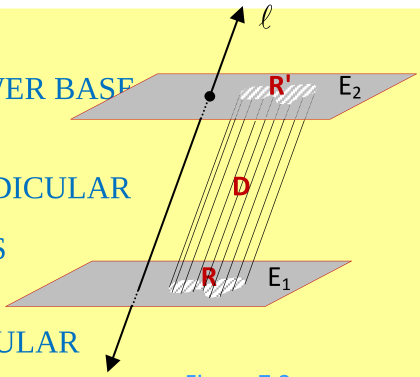


Figure 7.2

The line ℓ is called its **directrix** and the perpendicular distance between E_1 and E_2 is the **altitude of** D. If ℓ is perpendicular to E_1, then D is called a **right cylinder**, otherwise it is an **oblique cylinder**. If R is a circular region, then D is called a **circular cylinder**.

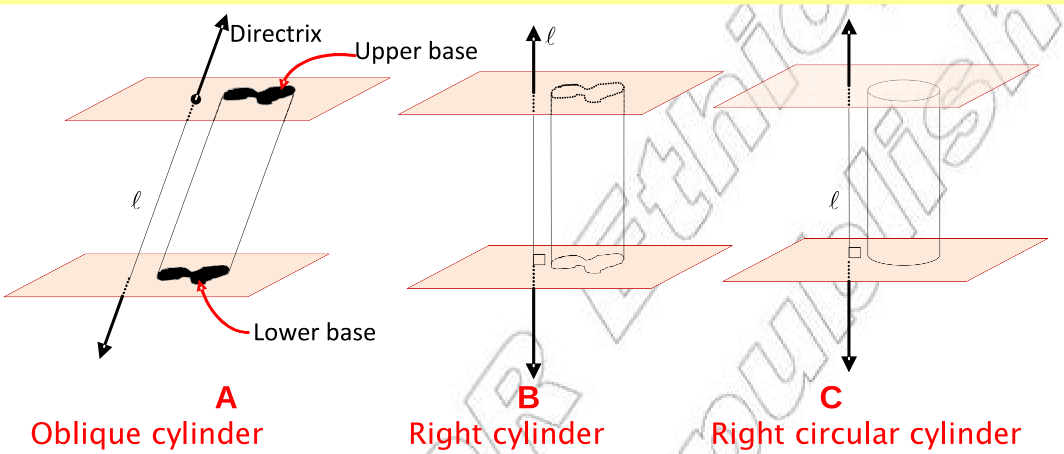


Figure 7.3

Let C be the bounding curve of the base region R. The union of all the elements $\overline{PP'}$ for which P belongs to C is called the **lateral surface** of the cylinder. The **total surface** is the union of the lateral surface and the bases of the cylinder.

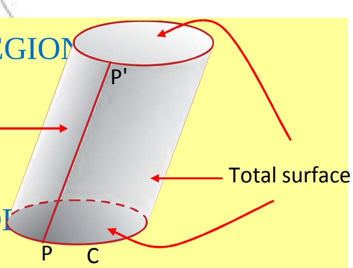


Figure 7.4

There are other familiar solid figures that are special cases of the solid looking figure D described above in Figure 7.2.

Definition 1.1

If R is a polygonal region, then D is called a **prism**.
If R is a parallelogram region, then D is a **parallelepiped**.
If R is a triangular region, then D is a **triangular prism**.
If R is a square region, then D is a **square prism**.
A **cube** is a square right prism whose altitude is equal to the length of the edge of the base.

Note:

IN THE PRISM SHOWN IN FIGURE 7.5

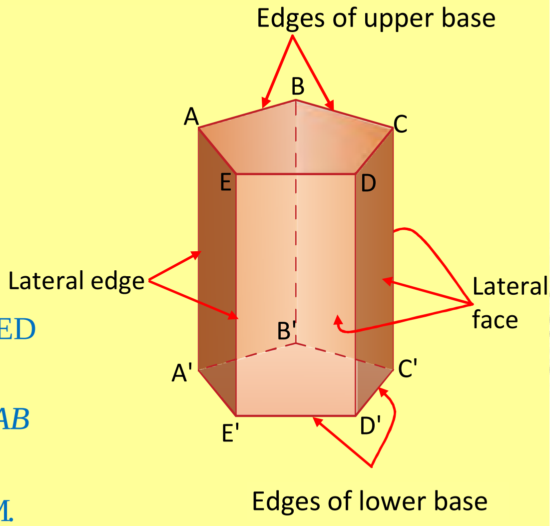


Figure 7.5

1 $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, $\overline{EA}$ ARE **edges** OF THE UPPER BASE.

$\overline{A'B'}$, $\overline{B'C'}$, $\overline{C'D'}$, $\overline{D'E'}$, $\overline{E'A'}$ ARE **edges** OF THE LOWER BASE.

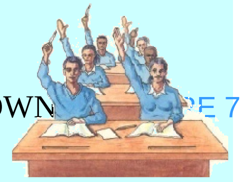
2 $\overline{AA'}$, $\overline{BB'}$, $\overline{CC'}$, $\overline{DD'}$, $\overline{EE'}$ ARE CALLED **lateral edges** OF THE PRISM.

3 THE PARALLELOGRAM REGIONS $ABB'A'$, $BCC'B'$, $AEE'A'$, $DCC'D'$, $EDD'E$ ARE CALLED **lateral faces** OF THE PRISM.

4 THE UNION OF THE LATERAL FACES OF A PRISM IS CALLED ITS **lateral surface**

5 THE UNION OF ITS LATERAL FACES AND ITS TWO BASES IS CALLED **total surface** OR SIMPLY ITS **surface**.

ACTIVITY 7.1



1 HOW MANY EDGES DOES THE BASE OF THE PRISM SHOWN IN FIGURE 7.5 HAVE? NAME THEM.

2 IDENTIFY EACH OF THE SOLIDS IN FIGURE 7.6 AS PRISM, CYLINDER, TRIANGULAR PRISM, RIGHT PRISM, PARALLELEPIPED, RECTANGULAR PARALLELEPIPED AND CUBE.

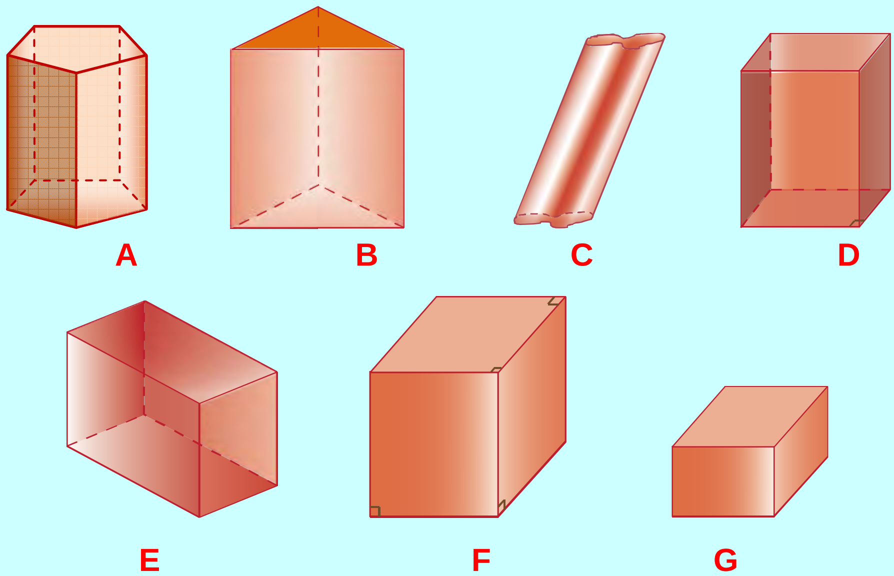


Figure 7.6

3 ARE THE LATERAL EDGES OF A PRISM EQUAL AND PARALLEL?

4 Using Figure 7.7, complete the following blank spaces to make true statements:

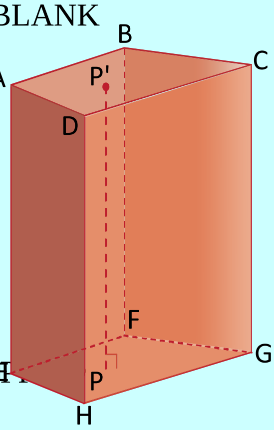


Figure 7.7

- **a** The figure is called a ______.
- **b** The region ABCD is called a __________.
- **c** $\overline{AE}$ and $\overline{CG}$ are called ______.
- **d** The region ADHE is called a ______ .
- **e** ______ is the altitude of the prism.
- **f** If *ABCD* were a parallelogram, then the prism would be called a ______.
- **g** If $\overline{AE}$ were perpendicular to the plane of the quadrilateral *EFGH*, then the prism would be called ______.

5 Consider a rectangular prism with dimensions of its base *l* and *w* and its height *h*. Determine:

a the base area **b** lateral surface area **c** total surface area

If we denote the lateral surface area of the prism by A_L, the area of the base by A_B, altitude h and the total surface area by A_T, then

$A_L = Ph$; where P is the perimeter of the base and h is the height of the prism.

$A_T = 2A_B + A_L$

Example 1 Find the lateral surface area of each of the following right prisms.

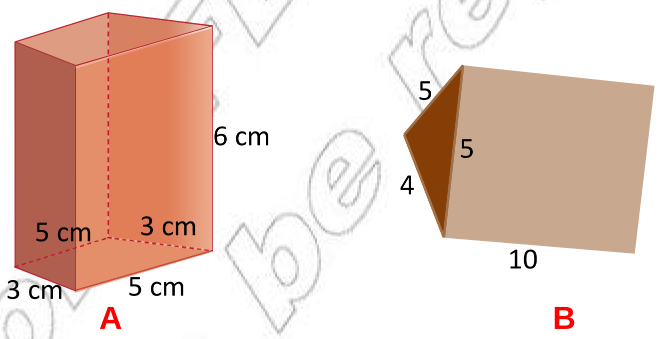


Figure 7.8

Solution:

- **a** $A_L = Ph = (3 + 5 + 3 + 5)\text{ cm} \times 6\text{ cm} = 16\text{ cm} \times 6\text{ cm} = 96\text{ cm}^2$
- **b** $A_L = Ph = (5 + 5 + 4) \times 10 = 14 \times 10 = 140\text{ units}^2$

Similarly, the lateral surface area (A_L) of a right circular cylinder is equal to the product of the circumference of the base and the altitude (h) of the cylinder. That is,

$A_L = 2\pi rh$, where r is the radius of the base of the cylinder.

THE TOTAL SURFACE AREA (A_T) IS EQUAL TO THE SUM OF THE AREAS OF THE BASES AND THE LATERAL SURFACE AR IS,

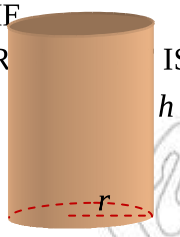


Figure 7.9

$$A_T = A_L + 2A_B$$

$$A_T = 2\pi r h + 2\pi r^2 = 2\pi r(h + r)$$

EXAMPLE 2 THE TOTAL SURFACE AREA OF A CIRCULAR CYLINDER IS 12π CM^2 AND THE ALTITUDE IS 1CM. FIND THE RADIUS OF THE BASE.

SOLUTION: $A_T = 2\pi r(h + r) \Rightarrow 12\pi = 2\pi r(1 + r) \Rightarrow 6 = r + r^2$

$r^2 + r - 6 = 0 \Rightarrow (r + 3)(r - 2) = 0 \Rightarrow r + 3 = 0$ OR $r - 2 = 0$

$\Rightarrow r = -3$ OR $r = 2$.

THEREFORE, THE RADIUS OF THE BASE IS 2 CM. (WHY?)

THE MEASUREMENT OF SPACE COMPLETELY ENCLOSED BY THE BOUNDING SURFACE OF A SOLID IS CALLED ITS **volume**.

THE VOLUME (V) OF ANY PRISM EQUALS THE PRODUCT OF ITS BASE AREA (A_B) AND ALTITUDE (h). THAT IS,

$$V = A_B h$$

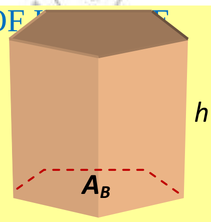


Figure 7.10

EXAMPLE 3 FIND THE TOTAL SURFACE AREA AND VOLUME OF THE FOLLOWING PRISM.

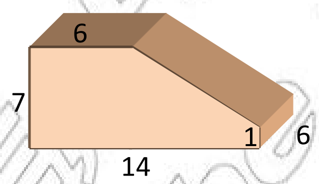


Figure 7.11

SOLUTION: TAKING THE BASE OF THE PRISM TO BE, AS SHADED IN THE FOLLOWING FIGURE, WE HAVE

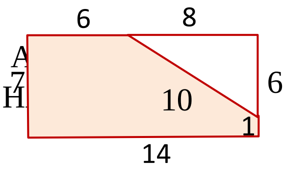


Figure 7.12

$$A_B = (7 \times 14) - \left(\frac{1}{2} \times 8 \times 6\right)$$

$$= 98 - 24 = 74 \text{ UNITS}^2$$

$$A_L = Ph = (7 + 6 + 10 + 14 + 1) \times 6$$

$$= 38 \times 6 = 228 \text{ UNITS}^2$$

$$A_T = A_L + 2A_B = 228 + 2 \times 74 = 376 \text{ UNITS}^2$$

$$V = A_B h = 74 \times 6 = 444 \text{ UNITS}^3$$

VOLUME OF A RIGHT CIRCULAR CYLINDER

THE VOLUME (V) OF A CIRCULAR CYLINDER IS EQUAL TO THE P[...] OF THE BASE AREA (A_B) AND ITS ALTITUDE (h). THAT IS,

$V = A_B h$

$V = \ r^2 h$, WHERE r IS THE RADIUS OF THE BASE.

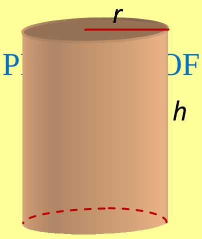


Figure 7.13

EXAMPLE 4 FIND THE VOLUME OF THE CYLINDER WHOSE BASE CIRCUMFERENCE IS 1 AND WHOSE LATERAL AREA IS 288 CM^2.

SOLUTION: $C = 2\ r \Rightarrow 12\ = 2\ r \Rightarrow r = 6$ CM

$A_L = 2\ r\,h$

$288\ \text{CM}^2 = 2\ \times 6\ \text{CM} \times h \Rightarrow 288\ \text{CM}^2 = 12\ \text{CM} \times h \Rightarrow h = 24$ CM

THEREFORE, $V = \ r^2 h = \ (6\ \text{CM})^2 \times 24\ \text{CM} = 36\ \text{CM}^2 \times 24\ \text{CM} = 864\ \text{CM}^3$

Exercise 7.1

1. THE ALTITUDE OF A RECTANGULAR PRISM IS 4 UNITS AND THE WIDTH AND LENGT[H] OF ITS BASE ARE 3 UNITS AND 2 UNITS RESPECTIVELY. FIND:

   A THE LATERAL SURFACE AREA B THE TOTAL SURFACE AREA C THE VOLUME

2. THE ALTITUDE OF THE RIGHT PENTAGONAL PRISM SHOWN IN FIGURE 7.14 IS 5 UNITS AND THE LENGTHS OF THE EDGES OF ITS BASE ARE 3, 4, 5, 6 AND 4 UNITS. FIND THE LATERA[L] AREA OF THE PRISM.

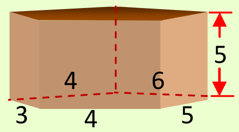


Figure 7.14

3. A LATERAL EDGE OF A RIGHT PRISM IS 6 CM AND THE PERIMETER OF ITS BASE IS 20 C[M]. FIND THE AREA OF ITS LATERAL SURFACE.

4. FIND THE LATERAL SURFACE AREA OF EACH OF THE SOLID FIGURES GIVEN IN FIGU[RE 7.15].

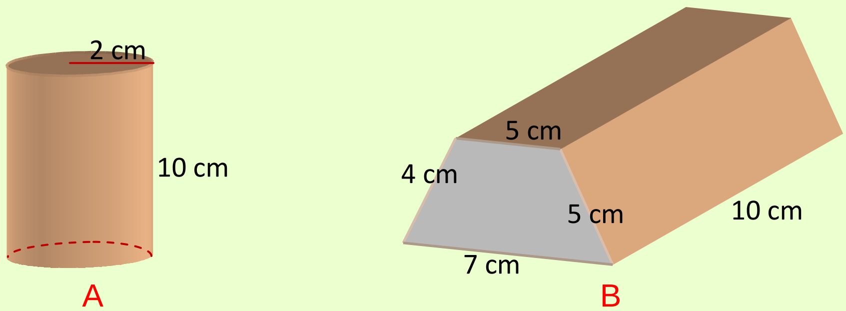


Figure 7.15

5 FIND THE PERIMETER OF THE BASE OF A RIGHT PRISM FOR WHICH THE AREA OF THE LATERAL SURFACE IS 180 UNITS^2 AND THE ALTITUDE IS 4 UNITS.

6 THE BASE OF A RIGHT PRISM IS AN EQUILATERAL TRIANGLE OF SIDE LENGTH 3 CM AND IT… SURFACES ARE RECTANGULAR REGIONS. IF ITS ALTITUDE IS 8 CM, THEN FIND:

A THE TOTAL SURFACE AREA OF THE PRISM **B** THE VOLUME OF THE PRISM.

7 IF THE DIMENSIONS OF A RIGHT RECTANGULAR PRISM ARE 7 CM, 9 CM AND 3 CM, TH…

A ITS TOTAL SURFACE AREA **B** ITS VOLUME

C THE LENGTH OF ITS DIAGONAL.

8 FIND THE TOTAL SURFACE AREA AND THE VOLUME OF EACH OF THE FOLLOWING SO…

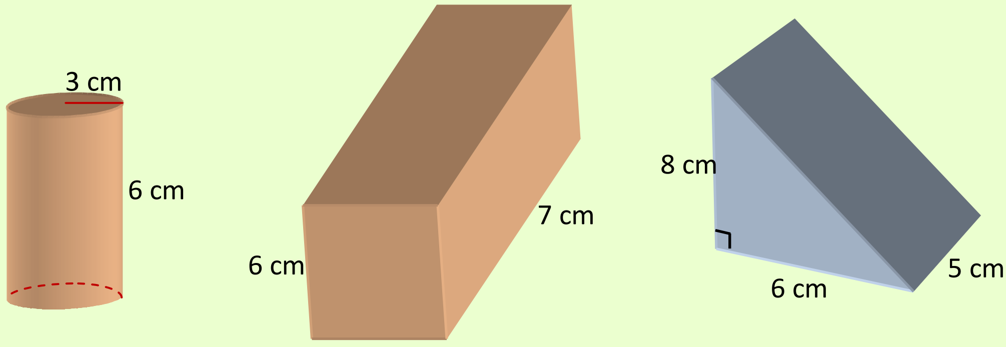


Figure 7.16

9 IF THE DIAGONAL OF A CUBE IS $\sqrt{12}$ CM, FIND THE AREA OF ITS LATERAL SURFACE.

10 THE RADIUS OF THE BASE OF A RIGHT CIRCULAR CYLINDER IS 2 CM AND ITS ALTITU… FIND:

A THE AREA OF ITS LATERAL SURFACE **B** THE TOTAL SURFACE AREA

C THE VOLUME.

11 SHOW THAT THE AREA OF THE LATERAL SURFACE OF A RIGHT CIRCULAR CYLINDER WHOSE ALTITUDE IS h AND WHOSE BASE HAS RADIUS r IS $2\pi rh$.

12 IMAGINE A CYLINDRICAL CONTAINER IN WHICH THE HEIGHT AND THE DIAMETER A… FIND EXPRESSIONS, IN TERMS OF ITS HEIGHT, FOR ITS

A TOTAL SURFACE AREA **B** VOLUME.

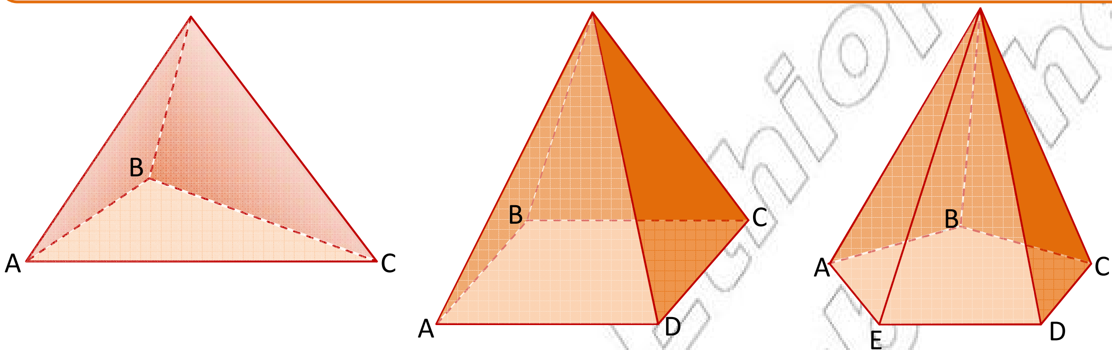
13 A CIRCULAR HOLE OF RADIUS 5 CM IS DRILLED THROUGH THE CENTRE OF A RIGHT CIRCULAR CYLINDER WHOSE BASE HAS RADIUS 6 CM AND WHOSE ALTITUDE IS 8 CM. FIND T… SURFACE AREA AND VOLUME OF THE RESULTING SOLID FIGURE.

7.2 PYRAMIDS, CONES AND SPHERES

DO YOU REMEMBER WHAT YOU LEARNT ABOUT PYRAMIDS, CONES AND SPHERES IN YO GRADES? CAN YOU GIVE SOME EXAMPLES OF PYRAMIDS, CONES AND SPHERES FROM REA

Definition 7.2

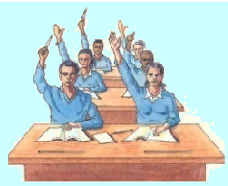
A pyramid is a solid figure formed when each vertex of a polygon is joined to the same point not in the plane of the polygon (See FIGURE 17).



TRIANGULAR PYRAMID **A** QUADRILATERAL PYRAMID **B** PENTAGONAL PYRAMID **C**

Figure 7.17

ACTIVITY 7.2



1 WHAT IS A REGULAR PYRAMID?

2 WHAT IS A TETRAHEDRON?

3 DETERMINE WHETHER EACH OF THE FOLLOWING STATEMENTS IS TRUE OR FALSE:

A THE LATERAL FACES OF A PYRAMID ARE TRIANGULAR REGIONS.

B THE NUMBER OF TRIANGULAR FACES OF A PYRAMID HAVING SAME VERTEX IS THE NUMBER OF EDGES OF THE BASE.

C THE ALTITUDE OF A CONE IS THE PERPENDICULAR DISTANCE FROM THE BASE VERTEX OF THE CONE.

4 USING FIGURE 7.18, COMPLETE THE FOLLOWING TO MAKE TRUE STATEMENTS.

A THE FIGURE IS CALLED A ______.

B THE REGION DVE IS CALLED A _______.

C THE REGION $ABCDEF$ IS CALLED ______.

D ______ IS THE ALTITUDE OF THE PYRAMID.

E $\overline{VE}$ AND $\overline{VF}$ ARE CALLED ______.

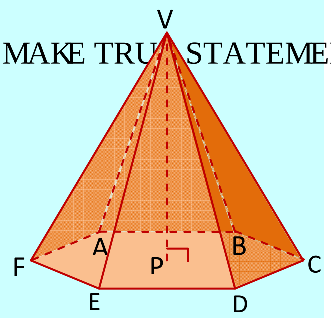


Figure 7.18

F SINCE *ABCDEF* IS A HEXAGONAL REGION, THE PYRAMID IS CALLED A ________.

5 DRAW A CONE AND INDICATE:

A ITS SLANT HEIGHT **B** ITS BASE **C** ITS LATERAL SURFACE.

THE **altitude** OF A PYRAMID IS THE LENGTH OF THE PERPENDICULAR FROM THE VERTEX TO CONTAINING THE BASE.

THE **slant height** OF A REGULAR PYRAMID IS THE ALTITUDE OF ANY OF ITS LATERAL FACE

Definition 7.3

The solid figure formed by joining all points of a circle to a point not on the plane of the circle is called a **cone**.

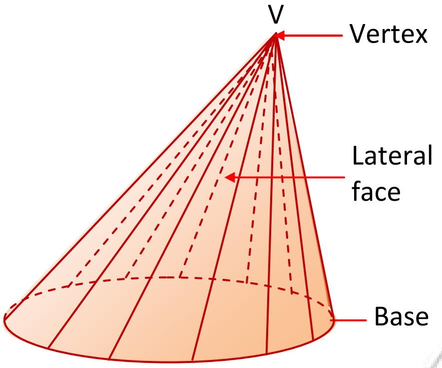


Figure 7.19

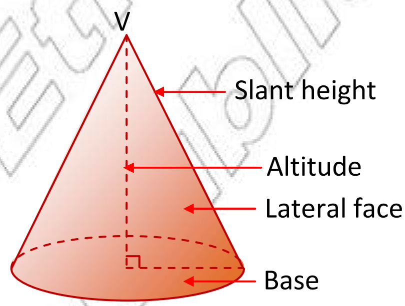


Figure 7.20

THE FIGURE SHOWN IN FIGURE 7.19 REPRESENTS A CONE. NOTE THAT THE CURVED SURFACE IS **lateral surface** OF THE CONE.

A **right circular cone** (SEE FIGURE 7.20) IS A CONE WITH THE FOOT OF ITS ALTITUDE AT CENTRE OF THE BASE. A LINE SEGMENT FROM THE VERTEX OF A CONE TO ANY POI BOUNDARY OF THE BASE (CIRCLE) IS CALLED THE **slant height**

ACTIVITY 7.3

1 CONSIDER A REGULAR SQUARE PYRAMID WITH BASE EDGE ... AND SLANT HEIGHT 5 CM.

A HOW MANY LATERAL FACES DOES IT HAVE?

B FIND THE AREA OF EACH LATERAL FACE.

C FIND THE LATERAL SURFACE AREA.

D FIND THE TOTAL SURFACE AREA.

2 TRY TO WRITE THE FORMULA FOR THE TOTAL SURFACE AREA OF A PYRAMID OR A C

Surface area

THE LATERAL SURFACE AREA OF A REGULAR PYRAMID IS EQUAL TO HALF THE PRODUCT HEIGHT AND THE PERIMETER OF THE BASE. THAT IS,

$$A_L = \frac{1}{2}P\ell,$$

WHERE A_L DENOTES THE LATERAL SURFACE AREA;
P DENOTES THE PERIMETER OF THE BASE;
ℓ DENOTES THE SLANT HEIGHT.

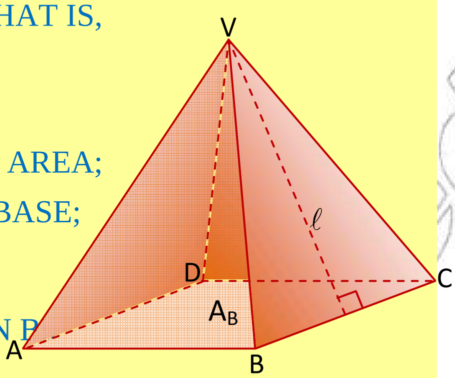


Figure 7.21

THE TOTAL SURFACE AREA (A_T) OF A PYRAMID IS GIVEN BY

$$A_T = A_B + A_L = A_B + \frac{1}{2}P\ell,$$

WHERE A_B IS AREA OF THE BASE.

EXAMPLE 1 A REGULAR PYRAMID HAS A SQUARE BASE WHOSE SIDE IS 4 CM LONG. THE LATERAL EDGES ARE 6 CM EACH.

A WHAT IS ITS SLANT HEIGHT? **B** WHAT IS THE LATERAL SURFACE AREA?

C WHAT IS THE TOTAL SURFACE AREA?

SOLUTION: CONSIDER FIGURE 7.22

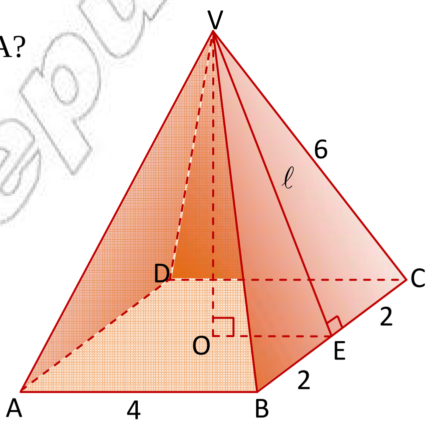


Figure 7.22

A $(VE)^2 + (EC)^2 = (VC)^2$

$\ell^2 + 2^2 = 6^2$

$\ell^2 = 32$

$\ell = 4\sqrt{2}$ CM

THEREFORE, THE SLANT HEIGHT IS $4\sqrt{2}$ CM

B THERE ARE 4 ISOSCELES TRIANGLES.

THEREFORE,

$$A_L = 4\times\frac{1}{2}BC \times VE = 4\left(\frac{1}{2}\times 4\times 4\sqrt{2}\right) = 32\sqrt{2}\ \text{CM}^2$$

$$\text{OR}\quad A_L = \frac{1}{2}P\ell = \frac{1}{2}(4+4+4+4)\,4\sqrt{2} = 8\times 4\sqrt{2} = 32\sqrt{2}\ \text{CM}^2$$

C $A_T = A_L + A_B = 32\sqrt{2} + 4\times 4$
$= 32\sqrt{2} + 16 = 16(2\sqrt{2}+1)\ \text{CM}^2$

THE LATERAL SURFACE AREA OF A RIGHT CIRCULAR CONE IS EQUAL TO HALF THE PRODUCT OF ITS SLANT HEIGHT AND THE CIRCUMFERENCE OF THE BASE. THAT IS,

$$A_L = \frac{1}{2}P\ell = \frac{1}{2}(2\ \mathrm{R})\ell = \ r\ell;$$

$$\ell = \sqrt{h^2 + r^2}$$

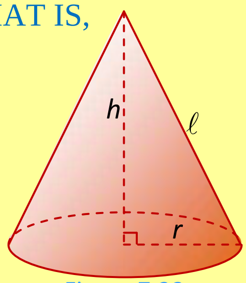


Figure 7.23

WHERE A_L DENOTES THE LATERAL SURFACE AREA, ℓ REPRESENTS THE SLANT HEIGHT, r STANDS FOR THE BASE RADIUS, AND h FOR THE ALTITUDE.

THE TOTAL SURFACE AREA (A_T) IS EQUAL TO THE SUM OF THE AREA OF THE BASE AND THE LATERAL SURFACE AREA. THAT IS,

$$A_T = A_L + A_B = \ r\ell + \ r^2 = \ r(\ell + r)$$

EXAMPLE 2 THE ALTITUDE OF A RIGHT CIRCULAR CONE IS 8 CM. IF THE RADIUS OF THE BASE IS 6 CM, THEN FIND ITS:

A SLANT HEIGHT **B** LATERAL SURFACE AREA **C** TOTAL SURFACE AREA.

SOLUTION: CONSIDER FIGURE 7.24

A $\ell = \sqrt{h^2 + r^2} = \sqrt{8^2 + 6^2} = \sqrt{100}$

$\ell = 10$ CM

B $A_L = \ r\ell = \ \times 6 \times 10 = 60\ $ CM^2

C $A_T = \ r(\ell + r) = \ \times 6\ (10 + 6) = 6\ \times 16$

$= 96\ $ CM^2

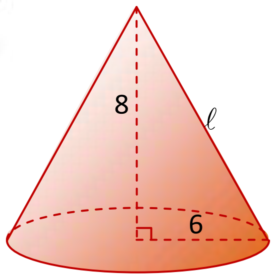


Figure 7.24

Volume

THE VOLUME OF ANY PYRAMID IS EQUAL TO ONE THIRD THE PRODUCT OF ITS BASE AREA AND ITS ALTITUDE. THAT IS,

$$V = \frac{1}{3}A_B h,$$

WHERE V DENOTES THE VOLUME, A_B THE AREA OF THE BASE AND h THE ALTITUDE.

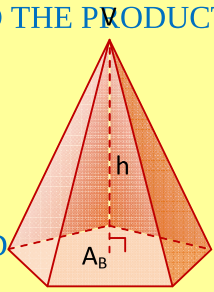


Figure 7.25

EXAMPLE 3 FIND THE VOLUME OF THE PYRAMID GIVEN IN EXAMPLE 1 ABOVE.

SOLUTION: HERE, WE NEED TO FIND THE ALTITUDE OF THE PYRAMID AS SHOWN BELOW

$$(VO)^2 + (OE)^2 = (VE)^2 \Rightarrow h^2 + 2^2 = (4\sqrt{2})^2$$
$$h^2 + 4 = 32$$
$$h^2 = 28 \Rightarrow h = 2\sqrt{7} \text{ CM}$$
$$V = \frac{1}{3} A_B h = \frac{1}{3} \times (4 \times 4) \times 2\sqrt{7} = \frac{32}{3}\sqrt{7} \text{ CM}^3$$

THE VOLUME OF A CIRCULAR CONE IS EQUAL TO ONE-THIRD OF THE PRODUCT OF ITS BASE AREA AND ITS ALTITUDE. THAT IS,

$$V = \frac{1}{3} A_B h = \frac{1}{3} r^2 h$$

WHERE V DENOTES THE VOLUME, r THE RADIUS OF THE BASE AND h THE ALTITUDE.

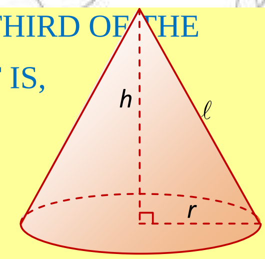


Figure 7.26

EXAMPLE 4 FIND THE VOLUME OF THE RIGHT CIRCULAR CONE GIVEN IN EXAMPLE 2 ABOVE.

SOLUTION: $V = \frac{1}{3} r^2 h = \frac{1}{3} (6)^2 \times 8 = 96 \text{ CM}^3$

EXAMPLE 5 FIND THE LATERAL SURFACE AREA, TOTAL SURFACE AREA AND THE VO THE FOLLOWING REGULAR PYRAMID AND RIGHT CIRCULAR CONE.

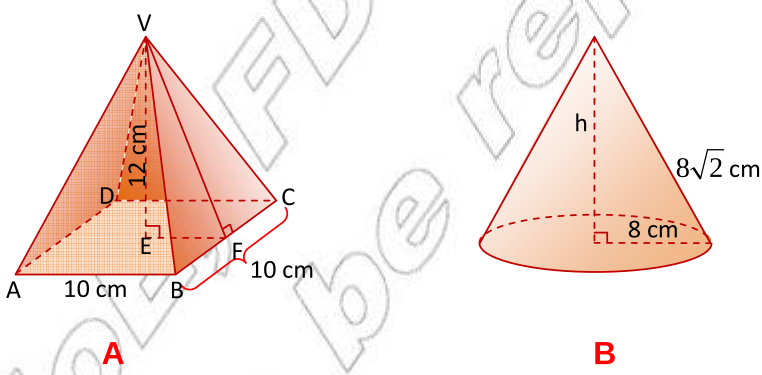


Figure 7.27

SOLUTION:

A TO FIND THE LATERAL SURFACE AREA, WE MUST FIND THE SLANT HEIGHT ℓ.

IN ΔVEF, WE HAVE,

$$(VE)^2 + (EF)^2 = (VF)^2 \Rightarrow 12^2 + 5^2 = (VF)^2$$
$$169 = (VF)^2 \Rightarrow VF = 13 \text{ CM}$$

THEREFORE, THE SLANT HEIGHT IS 13 CM.

$$\text{NOW, } A_B = \frac{1}{2}P\ell = \frac{1}{2}(10 + 10 + 10 + 10)13 = 260 \text{ CM}^2$$

$$A_T = A_L + A_B = 260 \text{ CM}^2 + 100 \text{ CM}^2 = 360 \text{ CM}^2$$

$$V = \frac{1}{3}A_B h = \frac{1}{3}\times 100\times 12 = 400 \text{ CM}^3.$$

B $\quad$ ALTITUDE : $h = \sqrt{\ell^2 - r^2} = \sqrt{(8\sqrt{2})^2 - 8^2} = \sqrt{128-64} = \sqrt{64} = 8$ CM

$$A_L = \ r\ell = \ \times 8\times 8\sqrt{2} = 64\sqrt{2} \ \text{ CM}^2$$

$$A_T = \ r(\ell + r) = 8 \ (8\sqrt{2} + 8) = 64 \ (\sqrt{2} + 1) \text{ CM}^2$$

$$V = \frac{1}{3} \ r^2 h = \frac{1}{3} \ (8)^2\times 8 = \frac{512}{3} \text{ CM}^3$$

Surface area and volume of a sphere

THE SPHERE IS ANOTHER SOLID FIGURE YOU STUDIED IN LOWER GRADES.

Definition 7.4

A sphere is a closed surface, all points of which are equidistant from a point called the centre.

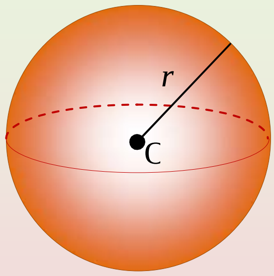


Figure 7.28

THE SURFACE AREA (A) AND THE VOLUME (V) OF A SPHERE OF RADIUS r ARE GIVEN BY

$$A = 4 \ r^2$$

$$V = \frac{4}{3} \ r^3$$

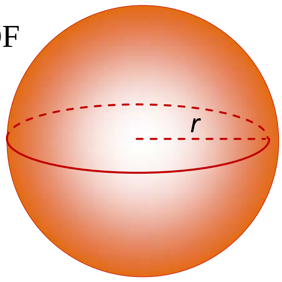


Figure 7.29

EXAMPLE 6 FIND THE SURFACE AREA AND VOLUME OF A SPHERICAL BALL WITH A DIAMETER OF 10 M.

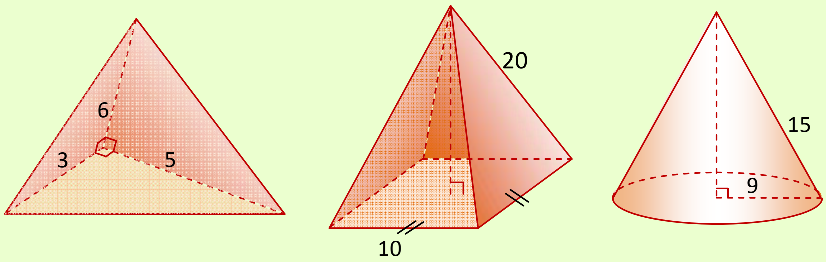
SOLUTION: WE KNOW THAT $d = 2r$ OR $r = \frac{d}{2}$ $\therefore r = \frac{10}{2} = 5$ M

$$A = 4 \ r^2 = 4 \ (5)^2 = 100 \ \text{M}^2$$

$$V = \frac{4}{3} \ r^3 = \frac{4}{3} \ (5)^3 = \frac{500}{3} \ \text{M}^3$$

Exercise 7.2

1 CALCULATE THE VOLUME OF EACH OF THE FOLLOWING SOLID FIGURES:



A B C

Figure 7.30

2 ONE EDGE OF A RIGHT SQUARE PYRAMID IS 6 CM LONG. IF THE LATERAL EDGE IS 8 CM, THEN FIND:

A ITS TOTAL SURFACE AREA **B** ITS VOLUME.

3 THE ALTITUDE OF A RIGHT EQUILATERAL TRIANGULAR PYRAMID IS 6 CM. IF ONE EDGE OF THE BASE IS 6 CM, THEN FIND:

A ITS TOTAL SURFACE AREA **B** ITS VOLUME.

4 A REGULAR SQUARE PYRAMID HAS ALL ITS EDGES 7 CM LONG. FIND:

A ITS TOTAL SURFACE AREA **B** ITS VOLUME

5 THE ALTITUDE AND RADIUS OF A RIGHT CIRCULAR CONE ARE 12 CM AND 5 CM RESPECTIVELY. FIND:

A ITS TOTAL SURFACE AREA **B** ITS VOLUME.

6 THE VOLUME OF A PYRAMID IS 240 CM^3. THE PYRAMID HAS A RECTANGULAR BASE WITH SIDES 6 CM BY 4 CM. FIND THE ALTITUDE AND LATERAL SURFACE AREA OF THE PY
PYRAMID HAS EQUAL LATERAL EDGES.

7 SHOW THAT THE VOLUME OF A REGULAR SQUARE PYRAMID WHOSE LATERAL FACES ARE EQUILATERAL TRIANGLES OF SIDE LENGTH s IS $\frac{s^3\sqrt{2}}{6}$.

8 THE LATERAL EDGE OF A REGULAR TETRAHEDRON IS 8 CM. FIND ITS ALTITUDE.

9 FIND THE VOLUME OF A CONE OF HEIGHT 12 CM AND SLANT HEIGHT 13 CM.

10 FIND THE VOLUME AND SURFACE AREA OF A SPHERICAL FOOTBALL WITH A RADIU

11 A GLASS IS IN THE FORM OF AN INVERTED CONE WHOSE BASE HAS A DIAMETER OF 10 CM. IF 0.1 LITRES OF WATER FILLS THE GLASS COMPLETELY, FIND THE DEPTH OF WATE

$\left(\text{TAKE} \approx \frac{22}{7}\right)$

12 A SOLID METAL CYLINDER WITH A LENGTH OF 24 CM AND RADIUS 2 CM IS MELTED D
FORM A SPHERE. WHAT IS THE RADIUS OF THE SPHERE?

7.3 FRUSTUMS OF PYRAMIDS AND CONES

IN THE PRECEDING SECTION, YOU HAVE STUDIED ABOUT PYRAMIDS AND CONES. YOU
STUDY THE SOLID FIGURE OBTAINED WHEN A PYRAMID AND A CONE ARE CUT BY A PLA
THE BASE AS SHOWN IN FIGURE 7.31.

LET E BE THE PLANE THAT CONTAINS THE BASE AND E' THE PLANE PARALLEL TO THE BASE THAT CU
THE PYRAMID AND CONE.

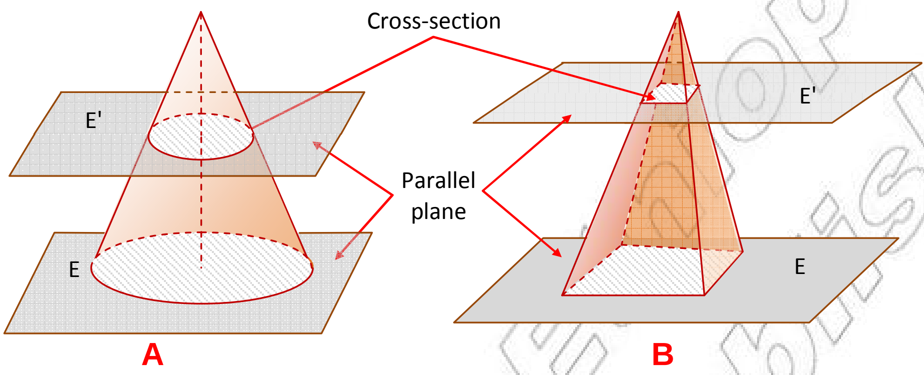


Figure 7.31

Definition 7.5

If a pyramid or a cone is cut by a plane parallel to the base, the intersection of the plane and the pyramid (or the cone) is called a horizontal cross-section of the pyramid (or the cone).

LET US NOW EXAMINE THE RELATIONSHIP BETWEEN THE BASE AND THE CROSS-SECTION

LET ΔABC BE THE BASE OF THE PYRAMID LYING IN THE PLANE E. LET h BE THE ALTITUDE OF THE PYRAMID AND LET $\Delta A'B'C'$ BE THE CROSS-SECTION AT A DISTANCE k UNITS FROM THE VERTEX.

LET D AND D' BE THE POINTS AT WHICH THE PERPENDICULAR FROM V MEETS E AND E', RESPECTIVELY.

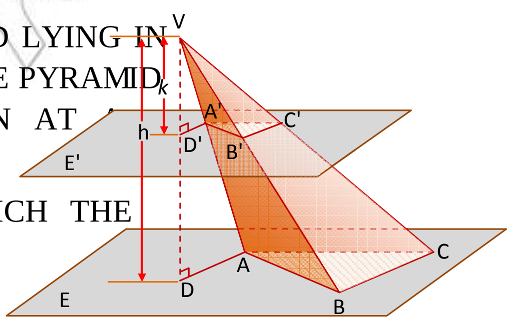


Figure 7.32

WE HAVE,

1 $\Delta VA'D' \sim \Delta VAD$.

THIS FOLLOWS FROM THE FACT THAT IF A PLANE INTERSECTS EACH OF TWO PARA
INTERSECTS THEM IN TWO PARALLEL LINES, AND AN APPLICATION OF THE AA
THEOREM. HENCE,

$$\frac{VA'}{VA} = \frac{VD'}{VD} = \frac{k}{h}$$

2 SIMILARLY, $\Delta VD'B' \sim \Delta VDB$ AND HENCE,

$$\frac{VB'}{VB} = \frac{VD'}{VD} = \frac{k}{h}$$

THEN, FROM **1** AND **2** AND THE SAS SIMILARITY THEOREM, WE GET,

3 $\Delta\, VA'B' \sim \Delta\, VAB$. THEREFORE, $\dfrac{A'B'}{AB} = \dfrac{VA'}{VA} = \dfrac{k}{h}$

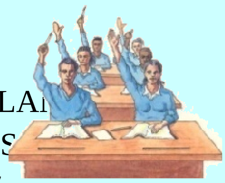
BY AN ARGUMENT SIMILAR TO THAT LEADING TO (3), WE HAVE

4 $\dfrac{B'C'}{BC} = \dfrac{k}{h}$ AND $\dfrac{A'C'}{AC} = \dfrac{k}{h}$

HENCE, BY THE SSS SIMILARITY THEOREM,

$$\Delta ABC \sim \Delta\, A'B'C'$$

ACTIVITY 7.4



IN THE PYRAMID SHOWN IN FIGURE 7.33, ΔABC IS EQUILATERAL. A PLANE PARALLEL TO THE BASE INTERSECTS THE LATERAL EDGES AT D, E AND F SUCH THAT $VE = \frac{1}{3}EB$.

A WHAT IS $\dfrac{VF}{VC}$?

B WHAT IS $\dfrac{EF}{BC}$?

C COMPARE THE AREAS OF ΔVEF AND ΔVBC AND OF ΔDEF AND ΔABC.

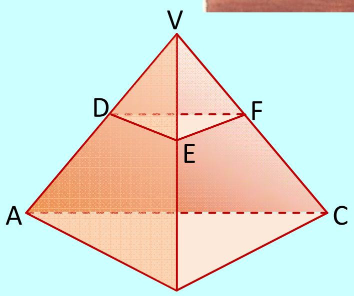
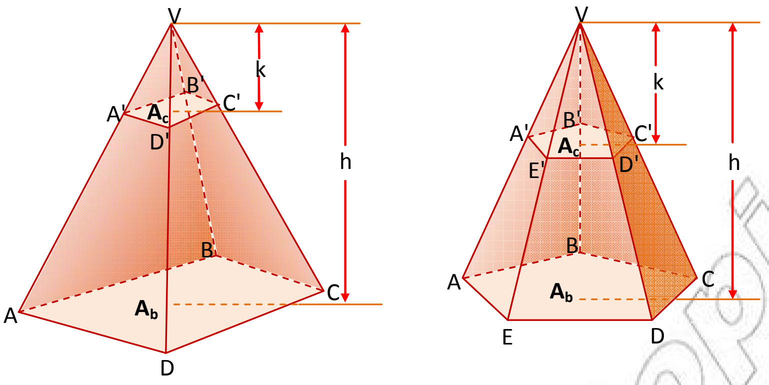


Figure 7.33

Theorem 7.1

In any pyramid, the ratio of the area of a cross-section to the area of the base is $\dfrac{k^2}{h^2}$ where h is the altitude of the pyramid and k is the distance from the vertex to the plane of the cross-section.



$$\frac{A_c}{A_b} = \frac{\text{AREA}(A'B'C'D')}{\text{AREA}(ABCD)} = \frac{k^2}{h^2} \qquad \frac{\text{AREA}(A'B'C'D'E')}{\text{AREA}(ABCDE)} = \frac{k^2}{h^2}$$

Figure 7.34

EXAMPLE 1 THE AREA OF THE BASE OF A PYRAMID IS 90 CM². THE ALTITUDE OF THE PYRAMID IS 12 CM. WHAT IS THE AREA OF A HORIZONTAL CROSS-SECTION 4 CM FROM THE

SOLUTION: LET A_c BE THE AREA OF THE CROSS-SECTION AND A_b THE BASE AREA.

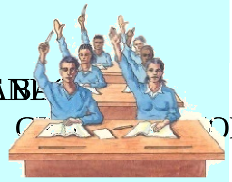
THEN, $\frac{A_c}{A_b} = \frac{k^2}{h^2} \Rightarrow \frac{A_c}{90} = \frac{4^2}{12^2}$

$\therefore \quad A_c = \frac{90\times 16}{144}$ CM² = 10 CM²

NOTE THAT SIMILAR PROPERTIES HOLD TRUE WHEN A CONE IS CUT BY A PLANE PARAL

Can you state them?

ACTIVITY 7.5

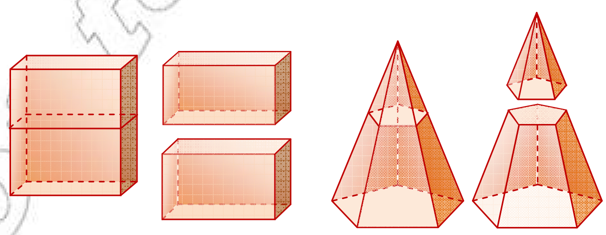


1 THE ALTITUDE OF A SQUARE PYRAMID IS 5 UNITS LONG AND A SIDE OF THE BA IS 4 UNITS LONG. FIND THE AREA OF A HORIZONTAL C ON AT A DISTANCE 2 UNITS ABOVE THE BASE.

2 THE AREA OF THE BASE OF A PYRAMID IS 64 CM². THE ALTITUDE OF THE PYRAMID IS 8 CM. WHAT IS THE AREA OF A CROSS-SECTION 2 CM FROM THE VERTEX?

3 THE RADIUS OF A CROSS-SECTION OF A CONE AT A DISTANCE 5 CM FROM THE BASE THE RADIUS OF THE BASE OF THE CONE IS 3 CM, FIND ITS ALTITUDE.

WHEN A PRISM IS CUT BY A PLANE PARALLEL TO THE BASE, EACH PART OF THE PRIS PRISM AS SHOWN IN FIGURE 7.35A.



A B

Figure 7.35

HOWEVER, WHEN A PYRAMID IS CUT BY A PLANE PARALLEL TO THE BASE, THE PART O BETWEEN THE VERTEX AND THE HORIZONTAL CROSS-SECTION IS AGAIN A PYRAMID WHE PART IS NOT A PYRAMID (AS SHOWN IN FIGURE 7.35B).

Frustum of a pyramid

Definition 7.6

A frustum of a pyramid is a part of the pyramid included between the base and a plane parallel to the base.

THE BASE OF THE PYRAMID AND THE CROSS-SECTION MADE BY THE PLANE PARALLEL TO THE bases of the frustum. THE OTHER FACES ARE CALLED lateral faces. THE TOTAL SURFACE OF A FRUSTUM IS THE SUM OF THE LATERAL SURFACE AND THE BASES.

THE altitude OF A FRUSTUM OF A PYRAMID IS THE PERPENDICULAR DISTANCE BETWEEN T

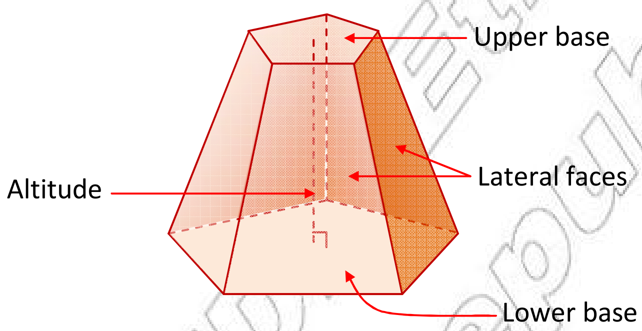


Figure 7.36

Note:

I THE LATERAL FACES OF A FRUSTUM OF A PYRAMID ARE TRAPEZIUMS.

II THE LATERAL FACES OF A FRUSTUM OF A REGULAR PYRAMID ARE ISOSCELES AND CONGR TRAPEZIUMS.

III THE SLANT HEIGHT OF A FRUSTUM OF A REGULAR PYRAMID IS THE ALTITUDE OF ANY ONE OF THE LATERAL FACES.

IV THE LATERAL SURFACE AREA OF A FRUSTUM OF A PYRAMID IS THE SUM OF THE AREAS OF THE LATERAL FACES.

Frustum of a cone

Definition 7.7

A frustum of a cone is a part of the cone included between the base and a horizontal cross-section made by a plane parallel to the base.

FOR A FRUSTUM OF A CONE, THE BASES ARE THE ... OF THE CONE AND THE CROSS-SECTION PARALLEL T... SE. THE **lateral surface** IS THE CURVED SURFACE T... MAKES UP THE FRUSTUM. THE ALTITUDE ... PERPENDICULAR DISTANCE BETWEEN THE BA...

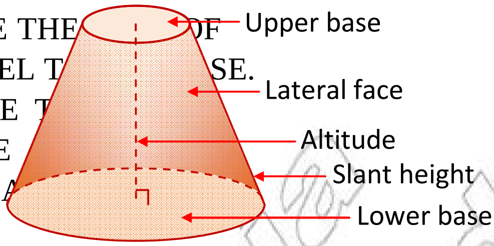


Figure 7.37

THE **slant height** OF A FRUSTUM OF A RIGHT CIRCULAR CONE IS THAT PART OF THE SLAN... THE CONE WHICH IS INCLUDED BETWEEN THE BASES.

CAN YOU NAME SOME OBJECTS WE USE IN REAL LIFE (AT HOME) THAT ARE FRUSTUMS... ARE A BUCKET AND A GLASS FRUSTUM OF CONES? DISCUSS.

EXAMPLE 2 THE LOWER BASE OF THE FRUSTUM OF A REGULAR PYRAMID IS A SQUARE 4 CM LONG. THE UPPER BASE IS 3 CM LONG. IF THE SLANT HEIGHT IS 6 CM, FIND ITS L... SURFACE AREA.

SOLUTION: AS SHOWN IN FIGURE 7.38, EACH LATERAL FACE IS A TRAPEZIUM, THE AREA O... EACH LATERAL FACE IS

$$A_L = \frac{1}{2}\times h(b_1 + b_2) = \frac{1}{2}\times 6(3+4) = 21\ \text{CM}^2$$

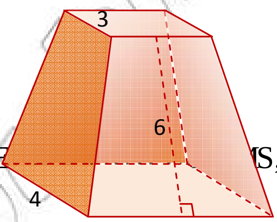


Figure 7.38

SINCE THE FOUR FACES ARE CONGRUENT ISOSCE... ...S, THE LATERAL SURFACE AREA IS

$$A_L = 4\times 21\ \text{CM}^2 = 84\ \text{CM}^2$$

EXAMPLE 3 THE LOWER BASE OF THE FRUSTUM OF A REGULAR PYRAMID IS A SQUARE OF SIDE s UNITS LONG. THE UPPER BASE IS s' UNITS LONG. IF THE SLA... HEIGHT OF THE FRUSTUM IS ℓ, THEN FIND THE LATERAL SURFACE AREA.

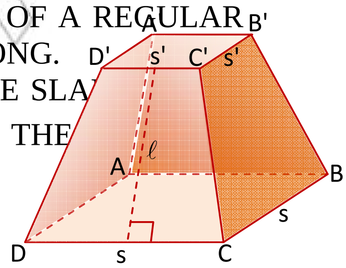


Figure 7.39

SOLUTION: FIGURE 7.39 REPRESENTS THE GIVEN PROBLEM. $ABCD$ IS A SQUARE s UNITS LONG. SIMILARLY $A'B'C'D'$ IS A SQUARE s' UNITS LONG.

LATERAL SURFACE AREA:

$$A_L = \text{AREA}(D'C'CD) + \text{AREA}(C'B'BC) + \text{AREA}(A'B'BA) + \text{AREA}(D'A'AD)$$

$$= \frac{1}{2}\ell(s+s') + \frac{1}{2}\ell(s+s') + \frac{1}{2}\ell(s+s') + \frac{1}{2}\ell(s+s')$$

$$A_L = \frac{1}{2}\ell(4s+4s') = 2\ell(s+s').$$

OBSERVE THAT $4s$ AND $4s'$ ARE THE PERIMETERS OF THE LOWER AND UPPER BASES, RESPECTIV...

IN GENERAL, WE HAVE THE FOLLOWING THEOREM:

Theorem 7.2

The lateral surface area (A_L) of a frustum of a regular pyramid is equal to half the product of the slant height (ℓ) and the sum of the perimeter (P) of the lower base and the perimeter (P') of the upper base. That is,

$$A_L = \frac{1}{2}\ell\,(P + P')$$

Group Work 7.1

CONSIDER THE FOLLOWING FIGURE.

1 FIND THE AREAS OF THE BASES.

2 FIND THE CIRCUMFERENCES OF THE BASES OF THE FRUSTUM, c_1 AND c_2.

3 FIND LATERAL SURFACE AREA OF THE BIGGER CONE.

4 FIND LATERAL SURFACE AREA OF THE SMALLER CONE.

5 FIND LATERAL SURFACE AREA OF THE FRUSTUM.

6 GIVE THE VOLUME OF THE FRUSTUM.

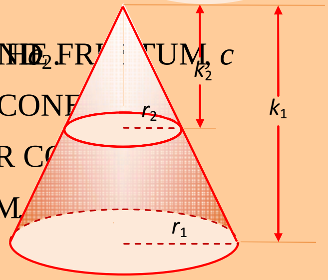


Figure 7.40

EXAMPLE 4 A FRUSTUM OF HEIGHT 4 CM IS FORMED FROM A RIGHT CIRCULAR CONE OF HEIGHT 8 CM AND BASE RADIUS 6 CM AS SHOWN IN FIGURE 7.41. CALCULATE THE LATERAL SURFACE AREA OF THE FRUSTUM.

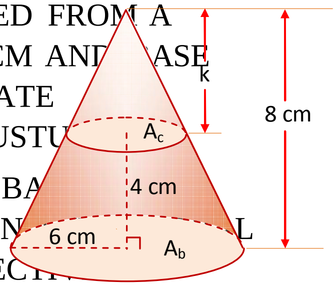


Figure 7.41

SOLUTION: LET A_b, A_c AND A_L STAND FOR AREA OF THE BASE OF THE CONE, AREA OF THE CROSS-SECTION AND LATERAL SURFACE AREA OF THE FRUSTUM, RESPECTIVELY.

$$\frac{\text{AREA OF CROSS-SECTION}}{\text{AREA OF THE BASE}} = \left(\frac{k}{h}\right)^2$$

$\frac{A_c}{A_b} = \left(\frac{4}{8}\right)^2$, SINCE $k = 8$ CM $- 4$ CM $= 4$ CM

$\frac{A_c}{36} = \frac{1}{4}$ (AREA OF THE BASE $= \pi r^2 = \pi \times 6^2 = 36\pi$)

$A_c = \frac{1}{4} \times 36 = 9$ CM^2

$A_c = (r')^2$, WHERE r' IS RADIUS OF THE CROSS-SECTION

$\therefore\ 9 = (r')^2 \Rightarrow r' = 3$ CM

SLANT HEIGHT OF THE BIGGER CONE IS:

$$\ell = \sqrt{h^2 + r^2} = \sqrt{8^2 + 6^2} = \sqrt{100} = 10 \text{ CM}$$

SLANT HEIGHT OF THE SMALLER CONE IS:

$$\ell' = \sqrt{k^2 + (r')^2} = \sqrt{4^2 + 3^2} = \sqrt{25} = 5 \text{ CM}$$

NOW THE LATERAL SURFACE AREA OF:

THE SMALLER CONE $r'\ell' = (3 \text{ CM}) \times 5 \text{ CM} = 15 \text{ CM}^2$

THE BIGGER CONE $r\ell = (6 \text{ CM}) \times 10 \text{ CM} = 60 \text{ CM}^2$.

HENCE, THE AREA OF THE LATERAL SURFACE OF THE FRUSTUM IS

$$A_L = 60 \text{ CM}^2 - 15 \text{ CM}^2 = 45 \text{ CM}^2.$$

THE LATERAL SURFACE (CURVED SURFACE) OF A FRUSTUM OF A CIRCULAR CONE IS A T
PARALLEL SIDES (BASES) HAVE LENGTHS EQUAL TO THE CIRCUMFERENCE OF THE BASES
AND WHOSE HEIGHT IS EQUAL TO THE HEIGHT OF THE FRUSTUM.

Theorem 7.3

For a frustum of a right circular cone with altitude h and slant height ℓ, if the circumferences of the bases are c and c', then the lateral surface area of the frustum is given by

$$A_L = \frac{1}{2}\ell(c + c') = \frac{1}{2}\ell(2 r + 2 r') = \ell (r + r')$$

EXAMPLE 5 A FRUSTUM FORMED FROM A RIGHT CIRCULAR CONE HAS BASE RADII OF 8 CM AND 12 CM AND SLANT HEIGHT OF 10 CM. FIND:

A THE AREA OF THE CURVED SURFACE

B THE AREA OF THE TOTAL SURFACE. (USE = 3.14).

SOLUTION:

A $A_L = \ell(r + r') = \times 10 \text{ CM} (8 + 12) \text{ CM} = 10 \text{ CM} \times 20 \text{ CM}$

$= 200 \text{ CM}^2 = 200 \times 3.14 \text{ CM}^2 = 628 \text{ CM}^2$

B AREA OF BASES:

$A_B = A_c + A_b = (r')^2 + r^2 = (8 \text{ CM})^2 + (12 \text{ CM})^2 = 64 \text{ CM}^2 + 144 \text{ CM}^2$

$= 208 \text{ CM}^2 \approx 208 \times 3.14 \text{ CM}^2 \approx 653 \text{ CM}^2$

TOTAL SURFACE AREA OF THE FRUSTUM:

$A_T = A_L + A_B \approx 628 \text{ CM}^2 + 653 \text{ CM}^2 = 1281 \text{ CM}^2$

EXAMPLE 6 THE AREA OF THE UPPER AND LOWER BASES OF A FRUSTUM OF A PYRAMID ARE 25 CM^2 AND 36 CM^2 RESPECTIVELY. IF ITS ALTITUDE IS 2 CM, THEN FIND THE ALTITUDE OF THE PYRAMID.

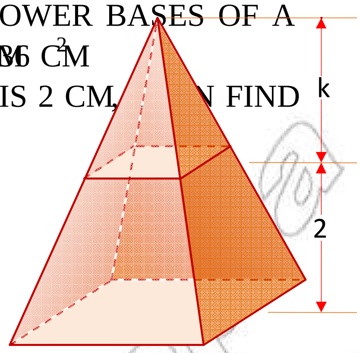


Figure 7.42

SOLUTION:

$$\frac{A_c}{A_b} = \left(\frac{k}{h}\right)^2 \Rightarrow \frac{25}{36} = \frac{k^2}{(2+k)^2}$$

$$\Rightarrow \frac{5}{6} = \frac{k}{2+k} \Rightarrow 6k = 5k + 10$$

$$\therefore \quad k = 10$$

THEREFORE, THE ALTITUDE OF THE PYRAMID IS 2 CM + 10 CM = 12 CM.

NOTE THAT THE UPPER AND LOWER BASES OF THE FRUSTUM OF A PYRAMID ARE SIMI AND THAT OF A CONE ARE SIMILAR CIRCLES.

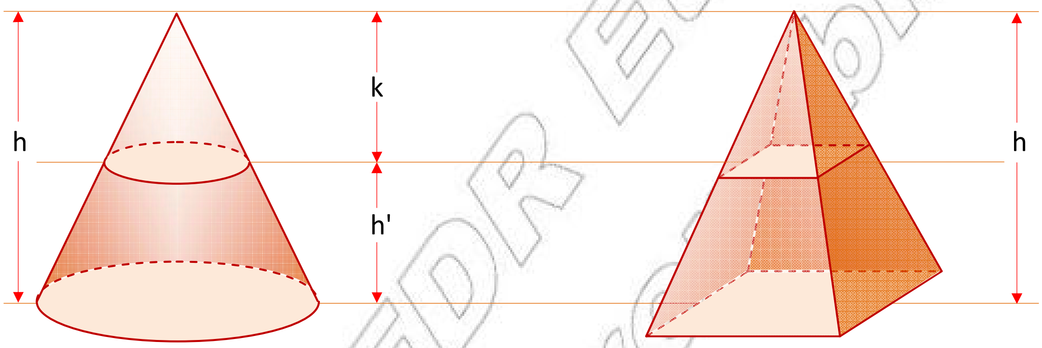


Figure 7.43

LET h = THE HEIGHT (ALTITUDE) OF THE COMPLETE CONE OR PYRAMID.

k = THE HEIGHT OF THE SMALLER CONE OR PYRAMID.

A = THE BASE AREA OF THE BIGGER CONE OR PYRAMID (LOWER BASE OF THE FRUST

A' = THE BASE AREA OF THE COMPLETING CONE OR PYRAMID (UPPER BASE OF THE F

$h' = h - k$ = THE HEIGHT OF THE FRUSTUM OF THE CONE OR PYRAMID.

V = THE VOLUME OF THE BIGGER CONE OR PYRAMID.

V' = THE VOLUME OF THE SMALLER CONE OR PYRAMID (UPPER PART).

V_f = THE VOLUME OF THE FRUSTUM

$V = \frac{1}{3}Ah$ AND $V' = \frac{1}{3}A'k$, CONSEQUENTLY THE VOLUME (V_f) OF THE FRUSTUM OF THE PYRAMID IS

$$V_f = V - V' = \frac{1}{3}Ah - \frac{1}{3}A'k = \frac{1}{3}(Ah - A'k)$$

USING THIS NOTION, WE SHALL GIVE THE FORMULA FOR FINDING THE VOLUME OF A F CONE OR PYRAMID AS FOLLOWS:

$$V_f = \frac{h'}{3}\left(A + A' + \sqrt{AA'}\right)$$

WHERE A IS THE LOWER BASE AREA, A' THE UPPER BASE AREA AND h' IS THE HEIGHT OF A FRUSTUM OF A CONE OR PYRAMID.

FROM THIS, WE CAN GIVE THE FORMULA FOR FINDING THE VOLUME OF A FRUSTUM OF A CONE IN TERMS OF r AND r' AS FOLLOWS:

$$V_f = \frac{\ }{3}h'\left(r^2 + (r')^2 + rr'\right)$$

WHERE r IS THE RADIUS OF THE BIGGER (THE LOWER BASE OF THE FRUSTUM) CONE AND r' IS THE RADIUS OF THE SMALLER CONE (UPPER BASE OF THE FRUSTUM).

EXAMPLE 7 A FRUSTUM OF A REGULAR SQUARE PYRAMID HAS HEIGHT 5 CM. THE UPPER BASE IS OF SIDE 2 CM AND THE LOWER BASE IS OF SIDE 6 CM. FIND THE VOLUME OF THE FRUSTUM.

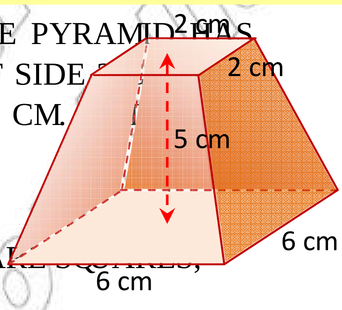


Figure 7.44

SOLUTION:

SINCE THE UPPER BASE AND LOWER BASE ARE SQUARES,

$A = (6\text{ CM})^2 = 36\text{ CM}^2$

$A' = (2\text{ CM})^2 = 4\text{ CM}^2$

$$V_f = \frac{h'}{3}\left(A + A' + \sqrt{AA'}\right) = \frac{5}{3}\left(36 + 4 + \sqrt{36 \times 4}\right)\text{CM}^3$$

$$= \frac{5}{3}(40 + 12)\text{ CM}^3 = \frac{5}{3} \times 52\text{ CM}^3 = \frac{260}{3}\text{ CM}^3$$

Exercise 7.3

1 THE LOWER BASE OF A FRUSTUM OF A REGULAR PYRAMID IS A SQUARE OF SIDE 6 CM, AND THE UPPER BASE HAS SIDE LENGTH 3 CM. IF THE SLANT HEIGHT IS 8 CM, FIND:

A ITS LATERAL SURFACE AREA **B** ITS TOTAL SURFACE AREA.

2 A CIRCULAR CONE WITH ALTITUDE h AND BASE RADIUS r IS CUT AT A HEIGHT $\frac{2}{3}$ OF THE WAY FROM THE BASE TO FORM A FRUSTUM OF A CONE. FIND THE VOLUME OF THE FRUSTUM.

3 THE AREAS OF BASES OF A FRUSTUM OF A PYRAMID ARE 25 CM^2 AND 49 CM^2. IF ITS ALTITUDE IS 3 CM, FIND ITS VOLUME.

4 THE SLANT HEIGHT OF A FRUSTUM OF A CONE IS 10 CM. IF THE RADII OF THE BASES ARE 8 CM AND 3 CM, FIND

A THE LATERAL SURFACE AREA **B** THE TOTAL SURFACE AREA

C THE VOLUME OF THE FRUSTUM.

5 A FRUSTUM OF A REGULAR SQUARE PYRAMID WHOSE LATERAL FACES ARE EQUILATERAL TRIANGLES OF SIDE 10 CM HAS ALTITUDE 5 CM. CALCULATE THE VOLUME OF THE FRUSTUM.

6 THE ALTITUDE OF A PYRAMID IS 10 CM. THE BASE IS A SQUARE WHOSE SIDES ARE EACH 6 CM LONG. IF A PLANE PARALLEL TO THE BASE CUTS THE PYRAMID AT A DISTANCE FROM THE VERTEX, THEN FIND THE VOLUME OF THE FRUSTUM FORMED.

7 THE BUCKET SHOWN IN FIGURE 7.45 IS IN THE FORM OF A FRUSTUM OF RIGHT CIRCULAR CONE. THE RADII OF THE BASES ARE 12 CM AND 20 CM, AND THE VOLUME IS 6000 CM^2. FIND THE

A HEIGHT **B** SLANT HEIGHT

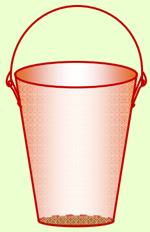


Figure 7.45

8 A FRUSTUM OF HEIGHT 12 CM IS FORMED FROM A RIGHT CIRCULAR CONE OF HEIGHT AND BASE RADIUS 8 CM. CALCULATE:

A THE LATERAL SURFACE AREA OF THE FRUSTUM

B THE TOTAL SURFACE AREA OF THE FRUSTUM

C THE VOLUME OF THE FRUSTUM.

9 A FRUSTUM IS FORMED FROM A REGULAR PYRAMID. LET THE PERIMETER OF THE LOWER BASE BE P, THE PERIMETER OF THE UPPER BASE BE P' AND THE SLANT HEIGHT BE ℓ. SHOW THAT THE LATERAL SURFACE AREA OF THE FRUSTUM IS

$$A_L = \frac{1}{2}\ell(P + P').$$

10 A FRUSTUM OF HEIGHT 5 CM IS FORMED FROM A RIGHT CIRCULAR CONE OF HEIGHT 10 CM AND BASE RADIUS 4 CM. CALCULATE:

A THE LATERAL SURFACE AREA **B** THE VOLUME OF THE FRUSTUM.

11 A FRUSTUM OF A REGULAR SQUARE PYRAMID HAS HEIGHT 2 CM. THE LATERAL FACES OF THE PYRAMID ARE EQUILATERAL TRIANGLES OF SIDE $2\sqrt{3}$ CM. FIND THE VOLUME OF THE FRUSTUM.

12 A CONTAINER IS IN THE SHAPE OF AN INVERTED FRUSTUM OF A RIGHT CIRCUL... CONE AS SHOWN IN FIGURE 7.46. IT HAS A CIRCULAR BOTTOM OF RADIUS 20 CM, A CIRCULAR TO... RADIUS 60 CM AND HEIGHT 40 CM. HOW MANY LITRES OF OIL COULD IT CONTAIN?

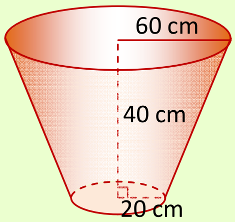


Figure 7.46

7.4 SURFACE AREAS AND VOLUMES OF COMPOSED SOLIDS

IN THE PRECEDING SECTIONS, YOU HAVE LEARNED HOW TO CALCULATE THE VOLUM... AND SURFACE AREA OF CYLINDERS, PRISMS, CONES, PYRAMIDS, SPHERES AND FRUSTUMS. IN THIS SE... WILL STUDY HOW TO FIND THE AREAS AND VOLUMES OF SOLIDS FORMED BY COMBINI... SOLID FIGURES.

ACTIVITY 7.6

1 GIVE THE FORMULA USED FOR:

A FINDING THE LATERAL SURFACE AREA OF A

I CYLINDER **II** PRISM **III** CONE **IV** PYRAMID

V SPHERE **VI** FRUSTUM OF A PYRAMID **VII** FRUSTUM OF A CONE

B FINDING THE VOLUME OF A

I CYLINDER **II** PRISM **III** CONE **IV** PYRAMID

V SPHERE **VI** FRUSTUM OF A PYRAMID **VII** FRUSTUM OF A CONE

2 IF THE DIAMETER OF A SPHERE IS HALVED, WHAT EFFECT DOES THIS HAVE ON ITS V... ITS SURFACE AREA?

3 WHAT IS THE RATIO OF THE VOLUME OF A SPHERE WHOSE RADIUS IS r UNITS TO THE CONE HAVING EQUAL RADIUS AND HEIGHT $2r$ UNITS?

CONSIDER THE FOLLOWING EXAMPLES.

EXAMPLE 1 A CANDLE IS MADE IN THE FORM OF A CIRCULAR CYLINDER OF RADIUS 4 CM AT THE BOTTOM AND A RIGHT CIRCULAR CONE OF ALTITUDE 3 CM, AS SHOWN IN FIGURE 7.47. IF THE OVERALL HEIGHT IS 12 CM, FIND THE TOTAL SURFACE AREA AND THE VOLUME OF THE CANDLE.

SOLUTION: SLANT HEIGHT OF THE CONE IS $\ell = \sqrt{3^2 + 4^2} = 5$ CM

THE TOTAL SURFACE AREA OF THE CANDLE IS THE SUM OF THE LATERAL SURFACE AREAS OF THE CONE, THE CYLIN… AND THE AREA OF THE BASE OF THE CYLINDER. THAT IS,

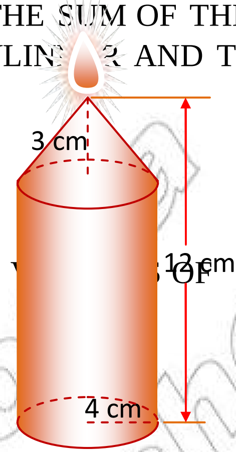


Figure 7.47

$$A_T = \ r\ell + 2\ rh + \ r^2 = \ (4)\,5 + 2\ (4)\,9 + \ (4)^2$$

$$= 20\ + 72\ + 16\ = 108\ \text{CM}^2$$

THE VOLUME OF THE CANDLE IS THE SUM OF THE V… OF THE CONE AND CYLINDER.

$$V_T = V_{cone} + V_{cylinder} = \frac{1}{3}\ r^2 h_{co} + \ r^2 h_{cy}$$

$$= \frac{1}{3}\ (4)^2 \times 3 + \ (4)^2 \times\ 9 = 16\ + 144\ = 160\ \text{CM}^3$$

EXAMPLE 2 THROUGH A RIGHT CIRCULAR CYLINDER WHOSE BASE RADIUS IS 10 CM AND WHOSE HEIGHT IS 1… DRILLED A TRIANGULAR PRISM HOLE WI… S EDGES 3 CM, 4 CM AND 5 CM AS SHOWN FIGURE 7.48. FIND THE TOTAL SURFACE AREA AN… THE REMAINING SOLID.

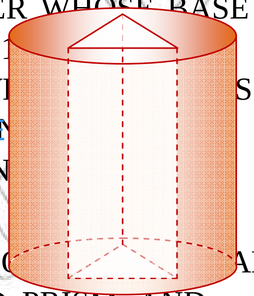


Figure 7.48

SOLUTION: THE TOTAL SURFACE AREA IS THE SUM O… AL SURFACE AREAS OF THE CYLINDER AND PRISM, AND THE BASE AREA OF THE CYLINDER, MINUS THE BASE AREA OF THE PRISM.

$$A_T = 2\ rh + ph + 2\ r^2 - 2\left(\frac{1}{2}ab\right)$$

$$= 2\ (10)\,12 + (3 + 4 + 5)\,12 + 2\ (10)^2 - 2\left(\frac{1}{2}\times 3\times 4\right)$$

$$= 240\ + 144 + 200\ - 12 = (440\ + 132)\ \text{CM}^2$$

THE VOLUME OF THE RESULTING SOLID IS THE DIFFERENCE BETWEEN THE VOL… CYLINDER AND PRISM.

$$V_T = V_{cy} - V_p = \ r^2 h - \frac{1}{2}abh = \ (10)^2 \times 12 - \frac{1}{2} \times 3 \times 4 \times 12$$

$$= 1200\ \text{CM}^3 - 72\ \text{CM}^3 = 24\,(50\ - 3)\ \text{CM}^3.$$

EXAMPLE 3 A CONE IS CONTAINED IN A CYLINDER … THEIR BASE RADIUS AND HEIGHT ARE THE … SHOWN IN FIGURE 7.49. CALCULATE THE VOL… OF THE SPACE INSIDE THE CYLINDER BUT … THE CONE.

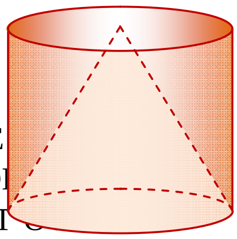


Figure 7.49

SOLUTION: THE REQUIRED VOLUME IS EQUAL TO THE DIFFERENCE BETWEEN THE VOLU CYLINDER AND THE CONE. THAT IS,

$$V = V_{cy} - V_{co} = \quad r^2h - \frac{1}{3} \quad r^2h = \frac{2}{3} \quad r^2h.$$

AS $r = h$, THEN $V = \frac{2}{3} \quad r^3$.

Group Work 7.2

1 A CYLINDRICAL TIN 8 CM IN DIAMETER CONTAINS W DEPTH OF 4 CM. IF A CYLINDRICAL WOODEN ROD 4 CM DIAMETER AND 6 CM LONG IS PLACED IN THE TIN IT FLOATS EXACTLY HALF SUBMERGED. WHAT IS THE NEW DEPTH OF WATER?

2 AN OPEN PENCIL CASE COMPRISES A CYLINDER OF LENGTH 20 CM AND RADIUS 2 C CONE OF HEIGHT 4 CM, AS SHOWN IN FIGURE 7.50. CALCULATE THE TOTAL SURFACE AREA A THE VOLUME OF THE PENCIL CASE.

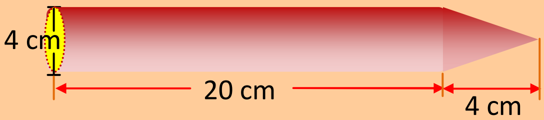
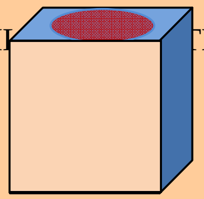


Figure 7.50

3 A BALL IS PLACED INSIDE A BOXINTO WHICH IT WI TLY.



IF THE RADIUS OF THE BALL IS 8 CM, CALCULATE:

I THE VOLUME OF THE BALL

II THE VOLUME OF THE BOX

Figure 7.51

4 AN ICE-CREAM CONSISTS OF A HEMISPHERE AND A CONE. CALCULATE ITS VOLUME AND TOTAL SURFAC

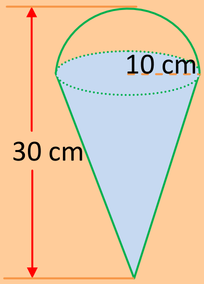


Figure 7.52

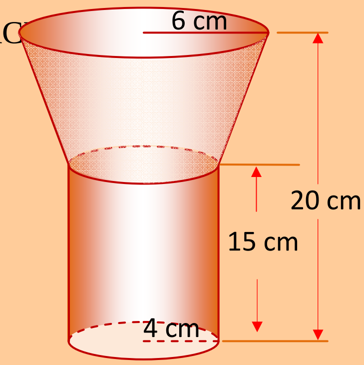


Figure 7.53

5 A TORCH 20 CM LONG IS IN THE FORM OF A RIGHT CIRCULAR CYLINDER OF HEIGHT 15 CM AND RADIUS 4 CM. JOINED TO IT IS A FRUSTUM OF A CONE OF RADIUS 6 CM. FIND THE VO THE TORCH.

Exercise 7.4

1 FIND THE VOLUME OF EACH OF THE FOLLOWING.

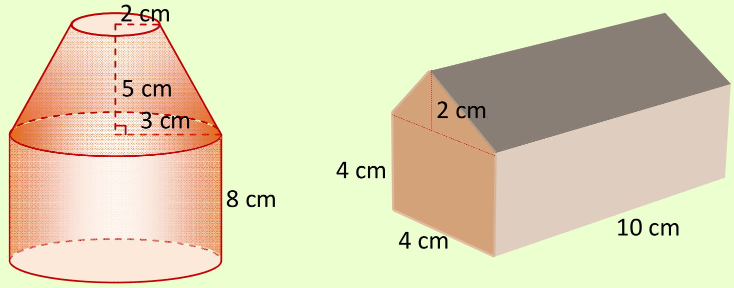


Figure 7.54

2 A STORAGE TANK IS IN THE FORM OF CYLINDER WITH ONE END HEMISPHERICAL, THE OTHER E BEING FLAT. THE DIAMETER OF THE CYLINDER IS 4 M AND THE OVERALL HEIGHT O 9 M. WHAT IS THE CAPACITY OF THE TANK?

3 AN IRON BALL 5 CM IN DIAMETER IS PLACED IN A CYLINDRICAL TIN OF DI WATER IS POURED INTO THE TIN UNTIL ITS DEPTH IS 6 CM. IF THE BALL IS NOW HOW FAR DOES THE WATER LEVEL DROP?

4 FROM A HEMISPHERICAL SOLID OF RADIUS 8 CM, A CONICAL PART IS REMOVE AS SHOWN IN FIGURE 7.55. FIND THE VOLUME AND THE TOTAL SURFACE AREA OF THE RESULTING S

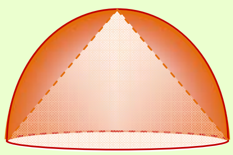


Figure 7.55

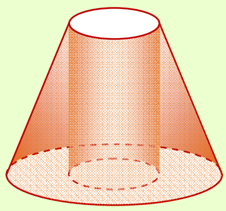


Figure 7.56

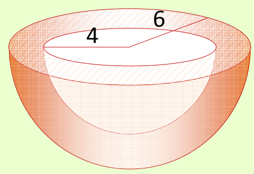


Figure 7.57

5 THE ALTITUDE OF A FRUSTUM OF A RIGHT CIRCULAR CONE IS 20 CM AND THE RADIUS OF ITS T BASE IS 6 CM. A CYLINDRICAL HOLE OF DIAMETER 4 CM IS DRILLED THROUGH THE THE CENTRE OF THE DRILL FOLLOWING THE AXIS OF THE CONE, LEAVING A SOLID FIGURE 7.56. FIND THE VOLUME AND THE TOTAL SURFACE AREA OF THE RESULTING S

6 FIGURE 7.57 SHOWS A HEMISPHERICAL SHELL. FIND THE VOLUME AND THE TOTAL SURFAC THE SOLID.

7 A CYLINDRICAL PIECE OF WOOD OF RADIUS 8 CM CM HAS A CONE OF THE SAME RADIUS SCOOPED DEPTH OF 9 CM. FIND THE RATIO OF THE VOLUM SCOOPED OUT TO THE VOLUME OF WOOD WHIC FIGURE 7.58)

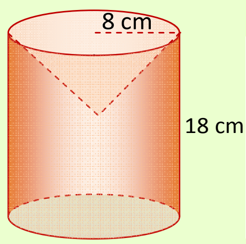


Figure 7.58

Key Terms

cone	lateral edge	regular pyramid
cross-section	lateral surface	slant height
cylinder	prism	sphere
frustum	pyramid	volume

Summary

Prism

$A_L = Ph$

$A_T = 2A_b + A_L$

$V = A_b\, h$

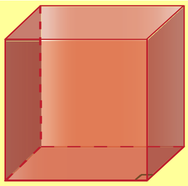


Figure 7.59

Right circular cylinder

$A_L = 2\ \ rh$

$A_T = 2\ \ r^2 + 2\ \ rh = 2\ \ r\,(r + h)$

$V = \ \ r^2 h$

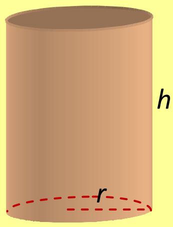


Figure 7.60

Regular pyramid

$$A_L = \frac{1}{2} P\ell$$

$$A_T = A_b + \frac{1}{2} P\ell$$

$$V = \frac{1}{3} A_b\, h$$

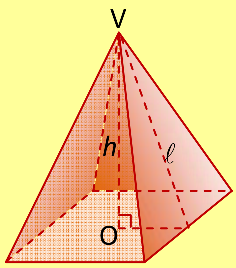


Figure 7.61

Right circular cone

$A_L = \ \ r\ell$

$A_T = \ \ r^2 + \ \ r\ell = \ \ r\,(r + \ell)$

$$V = \frac{1}{3}\ \ r^2 h$$

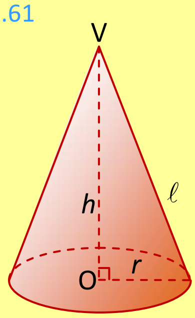


Figure 7.62

Sphere

$$A = 4\ \ r^2$$

$$V = \frac{4}{3}\ \ r^3$$

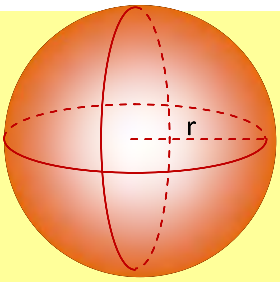


Figure 7.63

Frustum of a pyramid

$$A_L = \frac{1}{2}\ell\,(P + P')$$

$$A_T = \frac{1}{2}\ell\,(P + P') + A_b + A'_b$$

$$V = \frac{1}{3}h'\left(A_b + A'_b + \sqrt{A_b A'_b}\right)$$

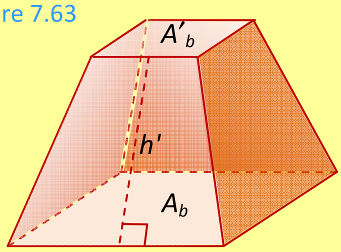


Figure 7.64

Frustum of a cone

$$A_L = \frac{1}{2}\ell(2\ \ r + 2\ \ r') = \ell\ \ (r + r')$$

$$A_T = \frac{1}{2}\ell(2\ \ r + 2\ \ r') +\ \ r^2 +\ \ (r')^2 = \ell\ \ (r + r') +\ \ (r^2 + r'^2)$$

$$V = \frac{1}{3}h'\ \ (r^2 + (r')^2 + rr')$$

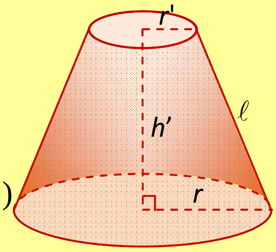


Figure 7.65

Review Exercises on Unit 7

1 FIND THE LATERAL SURFACE AREA AND VOLUME OF EACH OF THE FOLLOWING FIG

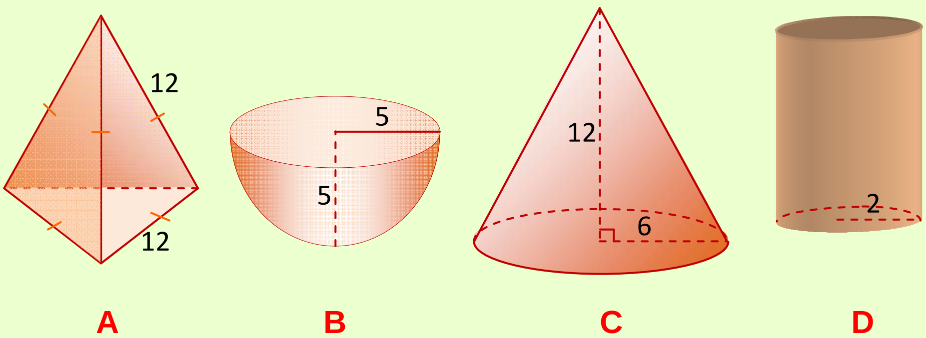


Figure 7.66

2 A LATERAL EDGE OF A RIGHT PRISM IS 6 CM AND THE PERIMETER OF ITS BASE IS 36 CM. FIND THE AREA OF ITS LATERAL SURFACE.

3 THE HEIGHT OF A CIRCULAR CYLINDER IS EQUAL TO THE RADIUS OF ITS BASE. FIND ITS TOTAL SURFACE AREA AND ITS VOLUME, GIVING YOUR ANSWER IN TERMS OF ITS RADIUS r

4 WHAT IS THE VOLUME OF A STONE IN AN EGYPTIAN PYRAMID WITH A SQUARE BASE OF SIDE 100 M AND A SLANT HEIGHT OF $50\sqrt{2}$ M FOR EACH OF THE TRIANGULAR FACES.

5 FIND THE TOTAL SURFACE AREA OF A REGULAR HEXAGONAL PYRAMID, GIVEN THAT A SIDE OF THE BASE IS 8 CM AND THE ALTITUDE IS 12 CM.

6 FIND THE AREA OF THE LATERAL SURFACE OF A RIGHT CIRCULAR CONE WHOSE ALTITUDE IS 8 CM AND BASE RADIUS 6 CM.

7 FIND THE TOTAL SURFACE AREA OF A RIGHT CIRCULAR CONE WHOSE ALTITUDE IS h AND BASE RADIUS IS r. (GIVE THE ANSWER IN TERMS OF r AND h)

8 WHEN A LUMP OF STONE IS SUBMERGED IN A RECTANGULAR WATER TANK WHOSE BASE IS 25 CM BY 50 CM, THE LEVEL OF THE WATER RISES BY 1 CM. WHAT IS THE VOLUME OF THE STONE?

9 A FRUSTUM WHOSE UPPER AND LOWER BASES ARE CIRCULAR REGIONS OF RADII 8 CM AND 6 CM RESPECTIVELY, IS 25 CM DEEP. (*See* FIGURE 7.67). FIND ITS VOLUME.

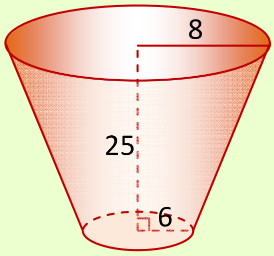


Figure 7.67

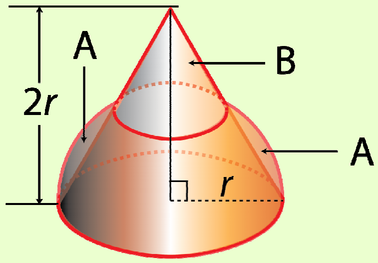


Figure 7.68

10 A CYLINDRICAL METAL PIPE OF OUTER DIAMETER 10 CM IS 2 CM THICK. WHAT IS THE DIAMETER OF THE HOLE? FIND THE VOLUME OF THE METAL IF THE PIPE IS 30 CM LONG.

11 A DRINKING CUP IN THE SHAPE OF FRUSTUM OF A CONE WITH BOTTOM DIAMETER 4 CM AND TOP DIAMETER 6 CM, CAN CONTAIN A MAXIMUM OF 80 CM^3 OF COFFEE. FIND THE HEIGHT OF THE CUP.

12 THE SLANT HEIGHT OF A CONE IS 16 CM AND THE RADIUS OF ITS BASE IS 4 CM. FIND THE AREA OF THE LATERAL SURFACE OF THE CONE AND ITS VOLUME.

13 THE RADIUS OF THE BASE OF A CONE IS 12 CM AND ITS VOLUME IS 720 CM^3. FIND ITS HEIGHT, SLANT HEIGHT, AND LATERAL SURFACE AREA.

14 IF THE RADIUS OF A SPHERE IS DOUBLED, WHAT EFFECT DOES THIS HAVE ON ITS VOLUME AND ITS SURFACE AREA?

15 IN FIGURE 7.68, A CONE OF BASE RADIUS r AND ALTITUDE $2r$ AND A HEMISPHERE OF RADIUS r WHOSE BASE COINCIDES WITH THAT OF THE CONE ARE SHOWN. A IS THE PART OF THE HEMISPHERE WHICH LIES OUTSIDE THE CONE AND B IS THE PART OF THE CONE LYING OUTSIDE THE HEMISPHERE. PROVE THAT THE VOLUME OF A IS EQUAL TO THE VOLUME OF B.

Table of Trigonometric Functions

	sin	cos	tan	cot	sec	csc	
0°	0.0000	1.0000	0.0000	.....	1.000	.....	**90°**
1°	0.0175	0.9998	0.0175	57.29	1.000	57.30	**89°**
2°	0.0349	0.9994	0.0349	28.64	1.001	28.65	**88°**
3°	0.0523	0.9986	0.0524	19.08	1.001	19.11	**87°**
4°	0.0698	0.9976	0.0699	14.30	1.002	14.34	**86°**
5°	0.0872	0.9962	0.0875	11.43	1.004	11.47	**85°**
6°	0.1045	0.9945	0.1051	9.514	1.006	9.567	**84°**
7°	0.1219	0.9925	0.1228	8.144	1.008	8.206	**83°**
8°	0.1392	0.9903	0.1405	7.115	1.010	7.185	**82°**
9°	0.1564	0.9877	0.1584	6.314	1.012	6.392	**81°**
10°	0.1736	0.9848	0.1763	5.671	1.015	5.759	**80°**
11°	0.1908	0.9816	0.1944	5.145	1.019	5.241	**79°**
12°	0.2079	0.9781	0.2126	4.705	1.022	4.810	**78°**
13°	0.2250	0.9744	0.2309	4.331	1.026	4.445	**77°**
14°	0.2419	0.9703	0.2493	4.011	1.031	4.134	**76°**
15°	0.2588	0.9659	0.2679	3.732	1.035	3.864	**75°**
16°	0.2756	0.9613	0.2867	3.487	1.040	3.628	**74°**
17°	0.2924	0.9563	0.3057	3.271	1.046	3.420	**73°**
18°	0.3090	0.9511	0.3249	3.078	1.051	3.236	**72°**
19°	0.3256	0.9455	0.3443	2.904	1.058	3.072	**71°**
20°	0.3420	0.9397	0.3640	2.747	1.064	2.924	**70°**
21°	0.3584	0.9336	0.3839	2.605	1.071	2.790	**69°**
22°	0.3746	0.9272	0.4040	2.475	1.079	2.669	**68°**
23°	0.3907	0.9205	0.4245	2.356	1.086	2.559	**67°**
24°	0.4067	0.9135	0.4452	2.246	1.095	2.459	**66°**
25°	0.4226	0.9063	0.4663	2.145	1.103	2.366	**65°**
26°	0.4384	0.8988	0.4877	2.050	1.113	2.281	**64°**
27°	0.4540	0.8910	0.5095	1.963	1.122	2.203	**63°**
28°	0.4695	0.8829	0.5317	1.881	1.133	2.130	**62°**
29°	0.4848	0.8746	0.5543	1.804	1.143	2.063	**61°**
30°	0.5000	0.8660	0.5774	1.732	1.155	2.000	**60°**
31°	0.5150	0.8572	0.6009	1.664	1.167	1.942	**59°**
32°	0.5299	0.8480	0.6249	1.600	1.179	1.887	**58°**
33°	0.5446	0.8387	0.6494	1.540	1.192	1.836	**57°**
34°	0.5592	0.8290	0.6745	1.483	1.206	1.788	**56°**
35°	0.5736	0.8192	0.7002	1.428	1.221	1.743	**55°**
36°	0.5878	0.8090	0.7265	1.376	1.236	1.701	**54°**
37°	0.6018	0.7986	0.7536	1.327	1.252	1.662	**53°**
38°	0.6157	0.7880	0.7813	1.280	1.269	1.624	**52°**
39°	0.6293	0.7771	0.8098	1.235	1.287	1.589	**51°**
40°	0.6428	0.7660	0.8391	1.192	1.305	1.556	**50°**
41°	0.6561	0.7547	0.8693	1.150	1.325	1.524	**49°**
42°	0.6691	0.7431	0.9004	1.111	1.346	1.494	**48°**
43°	0.6820	0.7314	0.9325	1.072	1.367	1.466	**47°**
44°	0.6947	0.7193	0.9667	1.036	1.390	1.440	**46°**
45°	0.7071	0.7071	1 .0000	1.000	1.414	1.414	**45°**
	cos	**sin**	**cot**	**tan**	**csc**	**sec**	

Table of Common Logarithms

N	0	1	2	3	4	5	6	7	8	9
1.0	0.0000	0.0043	0.0086	0.0128	0.0170	0.0212	0.0253	0.0294	0.0334	0.0374
1.1	0.0414	0.0453	0.0492	0.0531	0.0569	0.0607	0.0645	0.0682	0.0719	0.0755
1.2	0.0792	0.0828	0.0864	0.0899	0.0934	0.0969	0.1004	0.1038	0.1072	0.1106
1.3	0.1139	0.1173	0.1206	0.1239	0.1271	0.1303	0.1335	0.1367	0.1399	0.1430
1.4	0.1461	0.1492	0.1523	0.1553	0.1584	0.1614	0.1644	0.1673	0.1703	0.1732
1.5	0.1761	0.1790	0.1818	0.1847	0.1875	0.1903	0.1931	0.1959	0.1987	0.2014
1.6	0.2041	0.2068	0.2095	0.2122	0.2148	0.2175	0.2201	0.2227	0.2253	0.2279
1.7	0.2304	0.2330	0.2355	0.2380	0.2405	0.2430	0.2455	0.2480	0.2504	0.2529
1.8	0.2553	0.2577	0.2601	0.2625	0.2648	0.2672	0.2695	0.2718	0.2742	0.2765
1.9	0.2788	0.2810	0.2833	0.2856	0.2878	0.2900	0.2923	0.2945	0.2967	0.2989
2.0	0.3010	0.3032	0.3054	0.3075	0.3096	0.3118	0.3139	0.3160	0.3181	0.3201
2.1	0.3222	0.3243	0.3263	0.3284	0.3304	0.3324	0.3345	0.3365	0.3385	0.3404
2.2	0.3424	0.3444	0.3464	0.3483	0.3502	0.3522	0.3541	0.3560	0.3579	0.3598
2.3	0.3617	0.3636	0.3655	0.3674	0.3692	0.3711	0.3729	0.3747	0.3766	0.3784
2.4	0.3802	0.3820	0.3838	0.3856	0.3874	0.3892	0.3909	0.3927	0.3945	0.3962
2.5	0.3979	0.3997	0.4014	0.4031	0.4048	0.4065	0.4082	0.4099	0.4116	0.4133
2.6	0.4150	0.4166	0.4183	0.4200	0.4216	0.4232	0.4249	0.4265	0.4281	0.4298
2.7	0.4314	0.4330	0.4346	0.4362	0.4378	0.4393	0.4409	0.4425	0.4440	0.4456
2.8	0.4472	0.4487	0.4502	0.4518	0.4533	0.4548	0.4564	0.4579	0.4594	0.4609
2.9	0.4624	0.4639	0.4654	0.4669	0.4683	0.4698	0.4713	0.4728	0.4742	0.4757
3.0	0.4771	0.4786	0.4800	0.4814	0.4829	0.4843	0.4857	0.4871	0.4886	0.4900
3.1	0.4914	0.4928	0.4942	0.4955	0.4969	0.4983	0.4997	0.5011	0.5024	0.5038
3.2	0.5051	0.5065	0.5079	0.5092	0.5105	0.5119	0.5132	0.5145	0.5159	0.5172
3.3	0.5185	0.5198	0.5211	0.5224	0.5237	0.5250	0.5263	0.5276	0.5289	0.5302
3.4	0.5315	0.5328	0.5340	0.5353	0.5366	0.5378	0.5391	0.5403	0.5416	0.5428
3.5	0.5441	0.5453	0.5465	0.5478	0.5490	0.5502	0.5514	0.5527	0.5539	0.5551
3.6	0.5563	0.5575	0.5587	0.5599	0.5611	0.5623	0.5635	0.5647	0.5658	0.5670
3.7	0.5682	0.5694	0.5705	0.5717	0.5729	0.5740	0.5752	0.5763	0.5775	0.5786
3.8	0.5798	0.5809	0.5821	0.5832	0.5843	0.5855	0.5866	0.5877	0.5888	0.5899
3.9	0.5911	0.5922	0.5933	0.5944	0.5955	0.5966	0.5977	0.5988	0.5999	0.6010
4.0	0.6021	0.6031	0.6042	0.6053	0.6064	0.6075	0.6085	0.6096	0.6107	0.6117
4.1	0.6128	0.6138	0.6149	0.6160	0.6170	0.6180	0.6191	0.6201	0.6212	0.6222
4.2	0.6232	0.6243	0.6253	0.6263	0.6274	0.6284	0.6294	0.6304	0.6314	0.6325
4.3	0.6335	0.6345	0.6355	0.6365	0.6375	0.6385	0.6395	0.6405	0.6415	0.6425
4.4	0.6435	0.6444	0.6454	0.6464	0.6474	0.6484	0.6493	0.6503	0.6513	0.6522
4.5	0.6532	0.6542	0.6551	0.6561	0.6571	0.6580	0.6590	0.6599	0.6609	0.6618
4.6	0.6628	0.6637	0.6646	0.6656	0.6665	0.6675	0.6684	0.6693	0.6702	0.6712
4.7	0.6721	0.6730	0.6739	0.6749	0.6758	0.6767	0.6776	0.6785	0.6794	0.6803
4.8	0.6812	0.6821	0.6830	0.6839	0.6848	0.6857	0.6866	0.6875	0.6884	0.6893
4.9	0.6902	0.6911	0.6920	0.6928	0.6937	0.6946	0.6955	0.6964	0.6972	0.6981
5.0	0.6990	0.6998	0.7007	0.7016	0.7024	0.7033	0.7042	0.7050	0.7059	0.7067
5.1	0.7076	0.7084	0.7093	0.7101	0.7110	0.7118	0.7126	0.7135	0.7143	0.7152
5.2	0.7160	0.7168	0.7177	0.7185	0.7193	0.7202	0.7210	0.7218	0.7226	0.7235
5.3	0.7243	0.7251	0.7259	0.7267	0.7275	0.7284	0.7292	0.7300	0.7308	0.7316
5.4	0.7324	0.7332	0.7340	0.7348	0.7356	0.7364	0.7372	0.7380	0.7388	0.7396

5.5	0.7404	0.7412	0.7419	0.7427	0.7435		0.7443	0.7451	0.7459	0.7466	0.7474
5.6	0.7482	0.7490	0.7497	0.7505	0.7513		0.7520	0.7528	0.7536	0.7543	0.7551
5.7	0.7559	0.7566	0.7574	0.7582	0.7589		0.7597	0.7604	0.7612	0.7619	0.7627
5.8	0.7634	0.7642	0.7649	0.7657	0.7664		0.7672	0.7679	0.7686	0.7694	0.7701
5.9	0.7709	0.7716	0.7723	0.7731	0.7738		0.7745	0.7752	0.7760	0.7767	0.7774
6.0	0.7782	0.7789	0.7796	0.7803	0.7810		0.7818	0.7825	0.7832	0.7839	0.7846
6.1	0.7853	0.7860	0.7868	0.7875	0.7882		0.7889	0.7896	0.7903	0.7910	0.7917
6.2	0.7924	0.7931	0.7938	0.7945	0.7952		0.7959	0.7966	0.7973	0.7980	0.7987
6.3	0.7993	0.8000	0.8007	0.8014	0.8021		0.8028	0.8035	0.8041	0.8048	0.8055
6.4	0.8062	0.8069	0.8075	0.8082	0.8089		0.8096	0.8102	0.8109	0.8116	0.8122
6.5	0.8129	0.8136	0.8142	0.8149	0.8156		0.8162	0.8169	0.8176	0.8182	0.8189
6.6	0.8195	0.8202	0.8209	0.8215	0.8222		0.8228	0.8235	0.8241	0.8248	0.8254
6.7	0.8261	0.8267	0.8274	0.8280	0.8287		0.8293	0.8299	0.8306	0.8312	0.8319
6.8	0.8325	0.8331	0.8338	0.8344	0.8351		0.8357	0.8363	0.8370	0.8376	0.8382
6.9	0.8388	0.8395	0.8401	0.8407	0.8414		0.8420	0.8426	0.8432	0.8439	0.8445
7.0	0.8451	0.8457	0.8463	0.8470	0.8476		0.8482	0.8488	0.8494	0.8500	0.8506
7.1	0.8513	0.8519	0.8525	0.8531	0.8537		0.8543	0.8549	0.8555	0.8561	0.8567
7.2	0.8573	0.8579	0.8585	0.8591	0.8597		0.8603	0.8609	0.8615	0.8621	0.8627
7.3	0.8633	0.8639	0.8645	0.8651	0.8657		0.8663	0.8669	0.8675	0.8681	0.8686
7.4	0.8692	0.8698	0.8704	0.8710	0.8716		0.8722	0.8727	0.8733	0.8739	0.8745
7.5	0.8751	0.8756	0.8762	0.8768	0.8774		0.8779	0.8785	0.8791	0.8797	0.8802
7.6	0.8808	0.8814	0.8820	0.8825	0.8831		0.8837	0.8842	0.8848	0.8854	0.8859
7.7	0.8865	0.8871	0.8876	0.8882	0.8887		0.8893	0.8899	0.8904	0.8910	0.8915
7.8	0.8921	0.8927	0.8932	0.8938	0.8943		0.8949	0.8954	0.8960	0.8965	0.8971
7.9	0.8976	0.8982	0.8987	0.8993	0.8998		0.9004	0.9009	0.9015	0.9020	0.9025
8.0	0.9031	0.9036	0.9042	0.9047	0.9053		0.9058	0.9063	0.9069	0.9074	0.9079
8.1	0.9085	0.9090	0.9096	0.9101	0.9106		0.9112	0.9117	0.9122	0.9128	0.9133
8.2	0.9138	0.9143	0.9149	0.9154	0.9159		0.9165	0.9170	0.9175	0.9180	0.9186
8.3	0.9191	0.9196	0.9201	0.9206	0.9212		0.9217	0.9222	0.9227	0.9232	0.9238
8.4	0.9243	0.9248	0.9253	0.9258	0.9263		0.9269	0.9274	0.9279	0.9284	0.9289
8.5	0.9294	0.9299	0.9304	0.9309	0.9315		0.9320	0.9325	0.9330	0.9335	0.9340
8.6	0.9345	0.9350	0.9355	0.9360	0.9365		0.9370	0.9375	0.9380	0.9385	0.9390
8.7	0.9395	0.9400	0.9405	0.9410	0.9415		0.9420	0.9425	0.9430	0.9435	0.9440
8.8	0.9445	0.9450	0.9455	0.9460	0.9465		0.9469	0.9474	0.9479	0.9484	0.9489
8.9	0.9494	0.9499	0.9504	0.9509	0.9513		0.9518	0.9523	0.9528	0.9533	0.9538
9.0	0.9542	0.9547	0.9552	0.9557	0.9562		0.9566	0.9571	0.9576	0.9581	0.9586
9.1	0.9590	0.9595	0.9600	0.9605	0.9609		0.9614	0.9619	0.9624	0.9628	0.9633
9.2	0.9638	0.9643	0.9647	0.9652	0.9657		0.9661	0.9666	0.9671	0.9675	0.9680
9.3	0.9685	0.9689	0.9694	0.9699	0.9703		0.9708	0.9713	0.9717	0.9722	0.9727
9.4	0.9731	0.9736	0.9741	0.9745	0.9750		0.9754	0.9759	0.9763	0.9768	0.9773
9.5	0.9777	0.9782	0.9786	0.9791	0.9795		0.9800	0.9805	0.9809	0.9814	0.9818
9.6	0.9823	0.9827	0.9832	0.9836	0.9841		0.9845	0.9850	0.9854	0.9859	0.9863
9.7	0.9868	0.9872	0.9877	0.9881	0.9886		0.9890	0.9894	0.9899	0.9903	0.9908
9.8	0.9912	0.9917	0.9921	0.9926	0.9930		0.9934	0.9939	0.9943	0.9948	0.9952
9.9	0.9956	0.9961	0.9965	0.9969	0.9974		0.9978	0.9983	0.9987	0.9991	0.9996

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